

**Measuring Densities of Solids and Liquids Using Magnetic Levitation:
Fundamentals**

SUPPORTING INFORMATION

Katherine A. Mirica, Sergey S. Shevkoplyas, Scott T. Phillips, Malancha Gupta,
and George M. Whitesides*

Department of Chemistry & Chemical Biology, Harvard University, Cambridge, MA 02138

* Corresponding author E-mail: gwhitesides@gmwgroup.harvard.edu

General Methods

The NdFeB magnets (5 cm × 5 cm × 2.5 cm) were purchased from K&J Magnetics (www.kjmagnetics.com) and aligned on top of one another 4.5 cm apart within aluminum blocks. Similar magnets can also be obtained from Applied Magnets, www.magnet4less.com at a lower price. The strength of the magnetic field within the device was measured using a hand-held DC magnetometer (AlphaLab Inc, www.trifield.com). Calibrated density standards ($\pm 0.0002 \text{ g/cm}^3$ at 23°C) were purchased from American Density Materials (Stauton, VA; www.densitymaterials.com). Spherical polymer samples were purchased from McMaster-Carr (www.mcmaster.com). Polystyrene microspheres with precisely defined radii were supplied by Duke Scientific Corporation (www.dukescientific.com), Polysciences, Inc. (www.polysciences.com), and Spherotech (www.spherotech.com). All other samples and reagents were purchased from Sigma Aldrich (Atlanta, GA) and used without further purification. “Levitation height” of samples was measured using a ruler with millimeter-scaled marking. Helium pycnometry measurements were performed by Quantachrome Instruments for a fee on an Ultrapyc 1200e instrument.

Figure S0. A plot generated using numerical simulation using COMSOL Multiphysics showing the dependence of the z -component of the magnetic field B_z on the separation between magnets (h) for $h = 25, 35, 45, 55, 65, 75$ mm along the centerline between the two magnets.

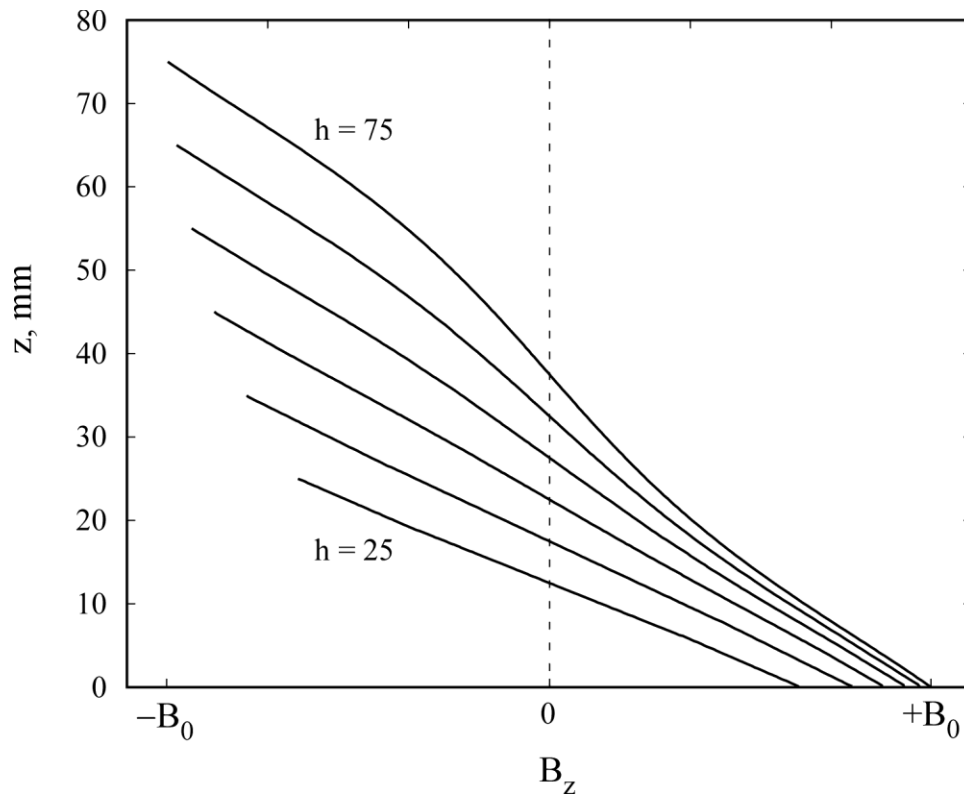


Figure S1. Photographs demonstrating levitation of a glass bead (density = 1.1500 ± 0.0002 g/cm³) in different aqueous paramagnetic solutions (1M MnCl₂, 1M MnBr₂, 1M CuSO₄, 1M GdCl₃, 1M FeCl₂).

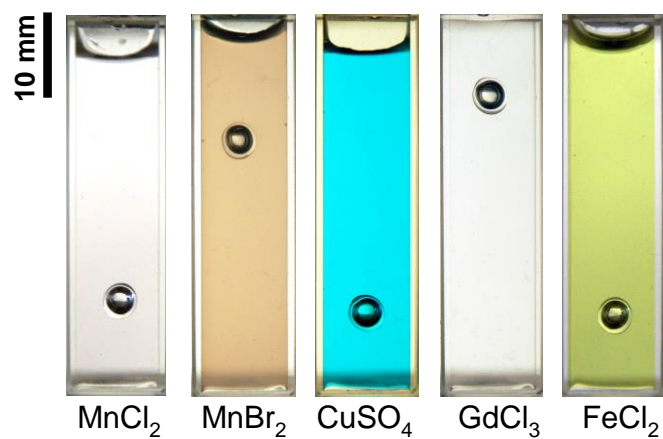


Figure S2. Net Volumetric Magnetic Susceptibilities of Common Diamagnetic Substances¹⁻⁷

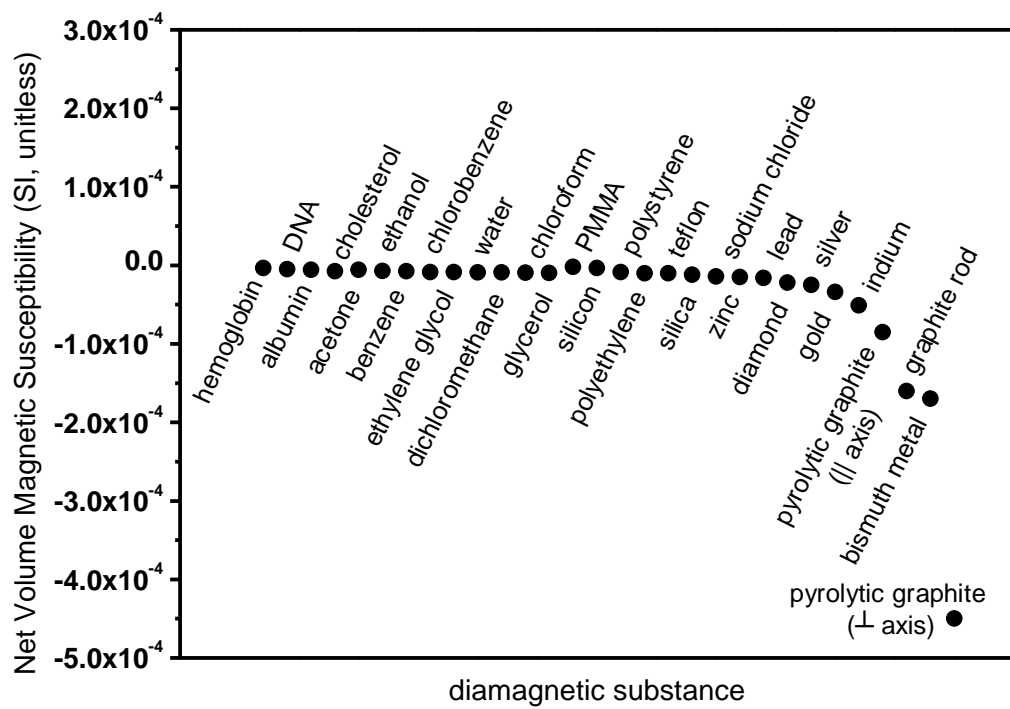


Figure S3. Deliberate misalignment of the container with the centerline between the magnets (red dotted line). Beads of different densities (from top to bottom: 1.0500, 1.0800, 1.1000, 1.1200, 1.1500 g/cm³) levitating in 1M MnCl₂ align with the centerline between the magnets regardless of the position of the container, as long as the centerline is accessible within the container (A and B). Inability of the beads to align with the centerline (C and D) does not result in significant change in the levitation height of the beads. Scale bar represents 10 mm.

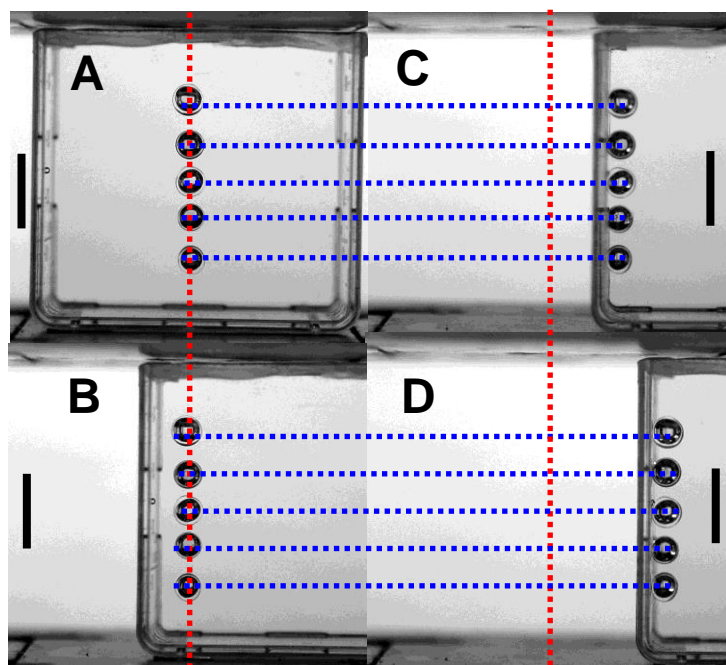


Figure S4. Effect of tilting the experimental set-up on height at which the objects levitate. A) Photographs of beads of different density (from top to bottom: 1.0500, 1.0800, 1.1000, 1.1200, 1.1500 g/cm³) levitating in 1 M MnCl₂ at different values of tilt angle θ . The dimensions of the container in which the beads levitate are 50 mm \times 30 mm \times 45 mm. The container spans the entire width and height of the magnets and is centered lengthwise between the magnets. B) Images in shown in panel A rotated by angle θ to emphasize the effect of tilting on the levitation height of the beads.

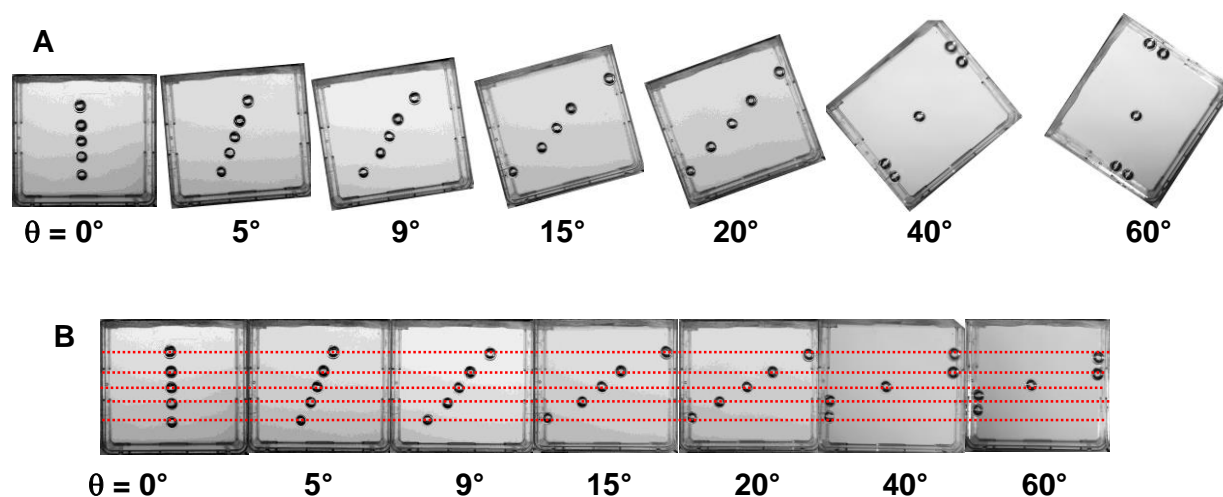
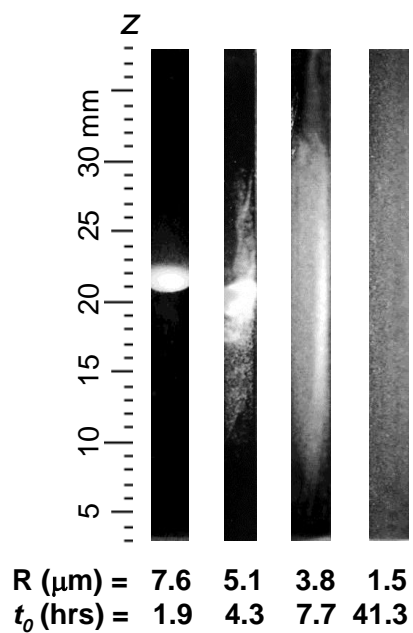


Figure S5. Photographs of polystyrene spheres of different radii levitating at t_0 in aqueous 500 mM MnCl_2 . Spheres of $R > \sim 7 \mu\text{m}$ form well defined clusters at z_0 , spheres of $\sim 2 \mu\text{m} < R < \sim 7 \mu\text{m}$ form diffuse clouds centered around z_0 , while spheres of $R < \sim 2 \mu\text{m}$ remain essentially uniformly dispersed throughout the solution.



Sources and Magnitude of Errors for Calculating Densities of Objects Using Eqn (8)

Experimental parameters and constants: B_0 , h , g , c , and T

We measured the magnitude of the magnetic field at the center on the surface of the bottom magnet with a magnetometer and found that in the configuration shown in **Fig. 1a** B_0 was (0.375 ± 0.003) Tesla.

The distance between the two magnets, h , remained constant in our experiments and was equal to (45 ± 0.5) mm.

The acceleration due to gravity, g , at the surface of the Earth ranges from $9.78 - 9.82$ m/s^2 , where the exact value depends on the latitude and other factors; we used the average value of 9.80 m/s^2 in our calculations and neglected this variation in our error analysis. The typical variation in g does not constitute a significant source of uncertainty in calculation of density using eqn 8.

We measured T using a thermometer with an accuracy of ± 1 °C.

We did not measure c directly—instead, we calculated it (eqn. S0A) based on the mass of the salt m (g) measured using an analytical balance with accuracy of $\delta m = 0.1 \times 10^{-3}$ g (as specified by the manufacturer of the analytical balance), the total volume of solution V (L) measured using a volumetric flask with accuracy of $\delta V = 0.08 \times 10^{-3}$ L (as specified by the manufacturer of the volumetric glassware), and the molecular weight of the salt M (g/mol) as provided by the vendor. We estimated the error associated with our calculation of concentration, δc , using eqn (S0B),⁸ and found it to be less than ± 0.002 M for the range of concentrations we used in this study.

$$c = \frac{m}{MV} \quad (\text{S0A})$$

$$\delta c = \sqrt{\left(\frac{1}{MV} \delta m\right)^2 + \left(-\frac{m}{MV^2} \delta V\right)^2} \quad (\text{S0B})$$

Magnetic Susceptibility of the Medium, χ_m

The magnetic susceptibility of the suspending medium, χ_m , depends on concentrations and magnetic properties of all the species present in solution. We calculated χ_m using eqn (S1), in which χ_p is the molar magnetic susceptibility of the paramagnetic salt (m^3/mol , units of SI), c is the concentration of the paramagnetic salt (mol/L), and -9×10^{-6} is the bulk magnetic susceptibility of water.^{9,10} The dependence of the molar magnetic susceptibilities of the paramagnetic salt χ_p on temperature follows the Curie-Weiss law given by eqn (S2), where T is the temperature of the medium (expressed in $^\circ\text{K}$), C_{CW} is the Curie constant ($\text{m}^3 \text{K} / \text{mol}$) and θ is the Weiss constant ($^\circ\text{K}$) of the paramagnetic salt.¹⁰ The dependence of magnetic susceptibility of the medium on temperature is then given by eqn (S3), in which we assumed that the magnetic properties of water do not change significantly with temperature.^{1,2}

$$\chi_m = \chi_p c - 9 \times 10^{-6} \quad (\text{S1})$$

$$\chi_p = \frac{C_{CW}}{T - \theta} \quad (\text{S2})$$

$$\chi_m = \frac{C_{CW}}{T - \theta} c - 9 \times 10^{-6} \quad (\text{S3})$$

In eqns (S1, S3), we neglected the contribution of the magnetic properties of gasses dissolved in the suspending medium. For example, air-saturated water at room temperature contains up to 0.3 mM of dissolved O_2 gas ($\chi_{\text{O}_2}^{\text{molar}} = 3449 \times 10^{-6}$)^{1,11} — the contribution of O_2 to

the magnetic susceptibility of the medium is $\chi_{O_2} = 1.3 \times 10^{-8}$, which is at least three orders of magnitude less than typical values of χ_m .

Density of the Paramagnetic Solution, ρ_m

The dependence of density of the medium ρ_m on the concentration of paramagnetic salt in solution and on temperature of the solution is given by eqn. (S4), where T is expressed in °C, $\rho_w(T)$ is the density of pure water ($W_0 = 999.65$, $W_1 = 2.0438 \times 10^{-1}$, $W_2 = -6.1744 \times 10^{-2}$), and A – F are empirical, dimensional parameters specific to the paramagnetic salt (for GdCl_3 : $A = 2.538 \times 10^2$, $B = -1.149 \times 10^{-1}$, $C = 1.386 \times 10^{-3}$, $D = -1.306 \times 10^{-1}$, $E = 0$, and $F = 0$; for MnCl_2 : $A = 1.022 \times 10^2$, $B = 4.966 \times 10^{-1}$, $C = -1.307 \times 10^{-2}$, $D = -3.659 \times 10^{-0}$, $E = -1.631 \times 10^{-1}$, and $F = 4.774 \times 10^{-3}$)¹².

$$\begin{aligned} \rho_m(T, c) &= \rho_w(T) + Ac + BcT + CcT^2 + Dc^{3/2} + Ec^{3/2}T + Fc^{3/2}T^2 = \\ &= W_0 + W_1T + W_2T^{3/2} + Ac + BcT + CcT^2 + Dc^{3/2} + Ec^{3/2}T + Fc^{3/2}T^2 \end{aligned} \quad (\text{S4})$$

Magnetic Susceptibility of the Sample, χ_s

The typical values of χ_s for diamagnetic substances are temperature-independent, small (about -1×10^{-6} ... -10×10^{-6}) and vary little from substance to substance (see **Fig. S2** for typical values of magnetic susceptibilities of common substances).^{1, 2, 9} We neglected these differences in calculations of densities based on the empirical estimates of α and β (eqn 8) from the calibration curves. For the measurements of density via the direct application of eqn (8), we neglected the magnetic susceptibility of the sample with respect to the magnetic susceptibility of the medium (that is, we set $\chi_s = 0$) and estimated the accuracy of this assumption as $\delta\chi_s = 10^{-5}$ (unitless).

This type of oversimplification and the estimation of uncertainty may not be general for all cases, especially when strongly diamagnetic ($|\chi_s| \gg 10^{-5}$) or slightly paramagnetic samples are levitated. In such cases, either an accurate value of χ_s should be used when calculating densities of eqn (8) or a larger margin of error assumed for this type of density measurement.¹³

Levitation Height, z_0

We measured the levitation heights of objects z_0 using a ruler—we estimate the precision of this measurement to be ± 0.5 mm.

Error Analysis

To estimate the error of our measurements of density using the calibration curves, we assume that we know parameters α and β in eqn (8) exactly, and consider ρ_s to be a function of only one variable z_0 ; we use eqn. (S5) to propagate the uncertainty in z_0 for these types of measurements.⁸ In eqn (S5), α is the calibration parameter from eqn (8) and δz_0 is ± 0.5 mm.

$$\delta\rho_s = \left| \frac{d\rho_s}{dz_0} \right| \delta z_0 = \alpha \delta z_0 \quad (\text{S5})$$

To estimate the error associated with the direct use of eqn (8) for measuring the density, we treat ρ_s as a function of several independent variables: B_0 , h , χ_s , T , c , and z_0 , each of which is a source of independent random error (eqn. S6).⁸ (Notice that ρ_m and χ_m are not independent parameters – they are calculated from T and c using eqns (S4) and (S3), respectively.)

$$\delta\rho_s = \sqrt{\left(\frac{\partial\rho_s}{\partial T} \delta T \right)^2 + \left(\frac{\partial\rho_s}{\partial c} \delta c \right)^2 + \left(\frac{\partial\rho_s}{\partial \chi_s} \delta \chi_s \right)^2 + \left(\frac{\partial\rho_s}{\partial z_0} \delta z_0 \right)^2 + \left(\frac{\partial\rho_s}{\partial h} \delta h \right)^2 + \left(\frac{\partial\rho_s}{\partial B_0} \delta B_0 \right)^2} \quad (\text{S6})$$

Eqn. (S7) summarizes eqn. (8) and eqns (S3) and (S4) in a form convenient for calculating the partial derivatives in eqn. (S6).

$$\rho_s = \left(\frac{4(\chi_s - \chi_m)B_0^2}{g\mu_o h^2} \right) z_0 + \left(\rho_m - \frac{2(\chi_s - \chi_m)B_0^2}{g\mu_o h} \right), \quad (\text{S7})$$

where:

$$\rho_m(T, c) = W_0 + W_1 T + W_2 T^{3/2} + A c + B c T + C c T^2 + D c^{3/2} + E c^{3/2} T + F c^{3/2} T^2$$

$$\chi_m = \frac{C_{CW}}{T - \theta} c - 9 \times 10^{-6}$$

The partial derivatives of ρ_m and χ_m with respect to T and c are needed for estimating eqn. (S6) and are given below (eqns S8A-B).

$$\frac{\partial \chi_m}{\partial T} = -C_{CW}c(T-\theta)^{-2} \quad (S8A)$$

$$\frac{\partial \rho_m}{\partial T} = W_1 + \frac{3}{2}W_2T^{1/2} + Bc + 2CcT + Ec^{3/2} + 2Fc^{3/2}T$$

$$\frac{\partial \chi_m}{\partial c} = \frac{C_{CW}}{T-\theta} \quad (S8B)$$

$$\frac{\partial \rho_m}{\partial c} = A + BT + CT^2 + \frac{3}{2}Dc^{1/2} + \frac{3}{2}ETc^{1/2} + \frac{3}{2}FT^2c^{1/2}$$

The partial derivatives of ρ_s with respect to B_0 , h , χ_s , T , c , and z_0 are given by eqns (S9A-F) below.

$$\frac{\partial \rho_s}{\partial T} = -\frac{4B_0^2}{g\mu_o h^2} \left(z_0 - \frac{h}{2} \right) \frac{\partial \chi_m}{\partial T} + \frac{\partial \rho_m}{\partial T},$$

where:

$$\frac{\partial \chi_m}{\partial T} = -C_{CW}c(T-\theta)^{-2} \quad (S9A)$$

$$\frac{\partial \rho_m}{\partial T} = W_1 + \frac{3}{2}W_2T^{1/2} + Bc + 2CcT + Ec^{3/2} + 2Fc^{3/2}T$$

$$\frac{\partial \rho_s}{\partial c} = -\frac{4B_0^2}{g\mu_o h^2} \left(z_0 - \frac{h}{2} \right) \frac{\partial \chi_m}{\partial c} + \frac{\partial \rho_m}{\partial c},$$

where:

$$\frac{\partial \chi_m}{\partial c} = \frac{C_{CW}}{T-\theta} \quad (S9B)$$

$$\frac{\partial \rho_m}{\partial c} = A + BT + CT^2 + \frac{3}{2}Dc^{1/2} + \frac{3}{2}ETc^{1/2} + \frac{3}{2}FT^2c^{1/2}$$

$$\frac{\partial \rho_s}{\partial \chi_s} = \frac{4B_0^2}{g\mu_o h^2} \left(z_0 - \frac{h}{2} \right) \quad (S9C)$$

$$\frac{\partial \rho_s}{\partial z_0} = \frac{4(\chi_s - \chi_m)B_0^2}{g\mu_o h^2} \quad (S9D)$$

$$\frac{\partial \rho_s}{\partial h} = \frac{2(\chi_s - \chi_m)B_0^2}{g\mu_o} (h^{-2} - 4z_0h^{-3}) \quad (S9E)$$

$$\frac{\partial \rho_s}{\partial B_0} = \frac{8(\chi_s - \chi_m)}{g \mu_o h^2} \left(z_0 - \frac{h}{2} \right) B_0 \quad (\text{S9F})$$

Finally, to estimate the error $\delta \rho_s$ in the measurement of density of the sample ρ_s one needs to substitute the expressions for the partial derivatives (eqns. S9A-F) into eqn. (S6) and evaluate the resulting expression using the values of parameters B_0 , h , χ_s , T , c , and z_0 , and the tolerances of these parameters $\delta B_0 = \pm 0.003$ (T), $\delta h = \pm 0.5 \times 10^{-3}$ (m), $\delta \chi_s = \pm 10^{-5}$ (unitless), $\delta T = 1$ ($^{\circ}\text{K}$), $\delta c = \pm 0.002$ (M), and $\delta z_0 = \pm 0.5 \times 10^{-3}$ (m), respectively. We provide two examples of successful application of this procedure below.

Dependence of $\delta \rho_s$ on δT .

We use eqn. (S6) (and eqns. (S9A-F)) to compare the effect of the uncertainty in temperature (± 1 $^{\circ}\text{C}$ vs. ± 10 $^{\circ}\text{C}$) on the accuracy of the measurement of density of polystyrene spheres levitating in 350 mM solution of MnCl_2 . This type of estimate may be appropriate for a user in a remote location with no immediate access to a thermometer for measuring temperature. For simplicity, we assume that all temperature-independent parameters and the value of B_0 are known *exactly*. **Table S1** summarizes the parameters used for and the result of this estimation.

Dependence of $\delta \rho_s$ on δB_0 .

Similarly, we use eqn. (S6) (and eqns. (S9A-F)) to compare the effect of $\delta B_0 = \pm 0.003$ T (typical accuracy established by using a magnetometer) vs. $\delta B_0 = 0.1$ T (a safe assumption for variation between NdFeB magnets supplied by various manufacturers) on the accuracy of the density measurement of polystyrene spheres levitating in 350 mM MnCl_2 . For simplicity, we

assume that all parameters except δB_0 are known exactly. **Table S2** summarizes the parameters used for and the result of this estimation – while the uncertainty in the measurement of density increases significantly from ± 0.0002 to 0.008 g/cm^3 , the measurement of density is still fairly accurate and may be useful in less demanding situations.

Table S1. Dependence of uncertainty in measurement of density $\delta\rho_s$ on uncertainty in δT .

| Parameter P | Description | Magnitude of P | δP | δP |
|--------------------------------|---|---|--------------------------------|--------------------------------|
| experimental parameters | | | | |
| B_0 | strength of magnetic field at the surface of the magnet | 0.375 T | known exactly | known exactly |
| h | distance between magnets | 45 mm | known exactly | known exactly |
| T | Temperature | 23 °C | ± 1 °C | ± 10 °C |
| c | Concentration of MnCl ₂ | 0.350 M | known exactly | known exactly |
| Unknowns | | | | |
| χ_s | bulk magnetic susceptibility of the sample | 0 (SI, unitless) | known exactly | known exactly |
| calculated parameters | | | | |
| $\rho_m(c, T)$ | density of paramagnetic medium | 1.0339 g/cm ³ | ± 0.0003 g/cm ³ | ± 0.0026 g/cm ³ |
| $\chi_m(c, T)$ | bulk magnetic susceptibility of the medium | $\pm 56 \times 10^{-6}$ | $\gg 1 \times 10^{-6}$ | $\gg 1 \times 10^{-6}$ |
| constants | | | | |
| g | acceleration due to gravity | 9.80 m/s ² | n/a | n/a |
| μ_0 | permeability of free space | $4\pi \times 10^{-6}$ N·A ⁻² | n/a | n/a |
| independent variable | | | | |
| z_0 | “levitation height” of the sample above the bottom magnet | 11.0 mm | known exactly | known exactly |
| dependent variable | | | | |
| ρ_s | density of sample | 1.0484 g/cm ³ | ± 0.0003 g/cm ³ | ± 0.003 g/cm ³ |

Table S2. Dependence of uncertainty in measurement $\delta\rho_s$ on uncertainty in δB_0 .

| Parameter P | Description | Magnitude of P | δP | δP |
|--------------------------------|---|---|--------------------------------|-------------------------------|
| experimental parameters | | | | |
| B_0 | strength of magnetic field at the surface of the magnet | 0.375 T | ± 0.003 T | ± 0.1 T |
| h | distance between magnets | 45 mm | known exactly | known exactly |
| T | Temperature | 23 °C | known exactly | known exactly |
| c | concentration of MnCl ₂ | 0.350 M | known exactly | known exactly |
| Unknowns | | | | |
| χ_s | bulk magnetic susceptibility of the sample | 0 (SI, unitless) | known exactly | known exactly |
| calculated parameters | | | | |
| $\rho_m(c, T)$ | density of paramagnetic medium | 1.0339 g/cm ³ | known exactly | known exactly |
| $\chi_m(c, T)$ | bulk magnetic susceptibility of the medium | $\pm 56 \times 10^{-6}$ | known exactly | known exactly |
| constants | | | | |
| g | acceleration due to gravity | 9.80 m/s ² | n/a | n/a |
| μ_0 | permeability of free space | $4\pi \times 10^{-6}$ N·A ⁻² | n/a | n/a |
| independent variable | | | | |
| z_0 | “levitation height” of the sample above the bottom magnet | 11.0 mm | known exactly | known exactly |
| dependent variable | | | | |
| ρ_s | density of sample | 1.0484 g/cm ³ | ± 0.0002 g/cm ³ | ± 0.008 g/cm ³ |

Table S3. Details of experimental parameters used for obtaining densities of solids using calibration curves in MnCl_2 . Table 2 compares the calculated densities from Tables S3-S5.

| Sample | $[\text{MnCl}_2]$ (mol/L) | z_0 (mm) | calibration curve used | Calculated ρ_s (g/cm^3) |
|-------------------------------------|------------------------------|----------------|----------------------------|---|
| glass beads | | | | |
| d = 1.0100 | 0.100 | 16.7 ± 0.5 | $z_0 = -3664\rho_s + 3717$ | 1.0099 ± 0.0002 |
| d = 1.1000 | 1.000 | 22.0 ± 0.5 | $z_0 = -247\rho_s + 294$ | 1.101 ± 0.002 |
| d = 1.1500 | 1.500 | 21.9 ± 0.5 | $z_0 = -160\rho_s + 206$ | 1.152 ± 0.003 |
| spherical polymers | | | | |
| polystyrene | 0.500 | 23.5 ± 0.5 | $z_0 = -449\rho_s + 493$ | 1.047 ± 0.001 |
| nylon 6/6 | 1.000 | 13.5 ± 0.5 | $z_0 = -247\rho_s + 294$ | 1.137 ± 0.002 |
| polymethylmethacrylate | 2.000 | 23.3 ± 0.5 | $z_0 = -117\rho_s + 162$ | 1.186 ± 0.004 |
| irregularly-shaped polymers | | | | |
| polystyrene | 0.500 | 22.0 ± 0.5 | $z_0 = -449\rho_s + 493$ | 1.049 ± 0.001 |
| poly(styrene-co-acrylonitrile) | 1.000 | 28.1 ± 0.5 | $z_0 = -247\rho_s + 294$ | 1.076 ± 0.002 |
| poly(styrene-co-methylmethacrylate) | 1.000 | 14.0 ± 0.5 | $z_0 = -247\rho_s + 294$ | 1.133 ± 0.002 |
| organic droplets | | | | |
| chlorobenzene | 1.000 | 18.5 ± 0.5 | $z_0 = -247\rho_s + 294$ | 1.115 ± 0.003 |
| 2-nitrotoluene | 1.000 | 5.2 ± 0.5 | $z_0 = -247\rho_s + 294$ | 1.169 ± 0.003 |
| dichloromethane | 3.000 | 19.5 ± 0.5 | $z_0 = -80.8\rho_s + 126$ | 1.324 ± 0.006 |
| 3-bromotoluene | 3.000 | 12.5 ± 0.5 | $z_0 = -80.8\rho_s + 126$ | 1.405 ± 0.006 |
| chloroform | 3.000 | 6.5 ± 0.5 | $z_0 = -80.8\rho_s + 126$ | 1.479 ± 0.007 |

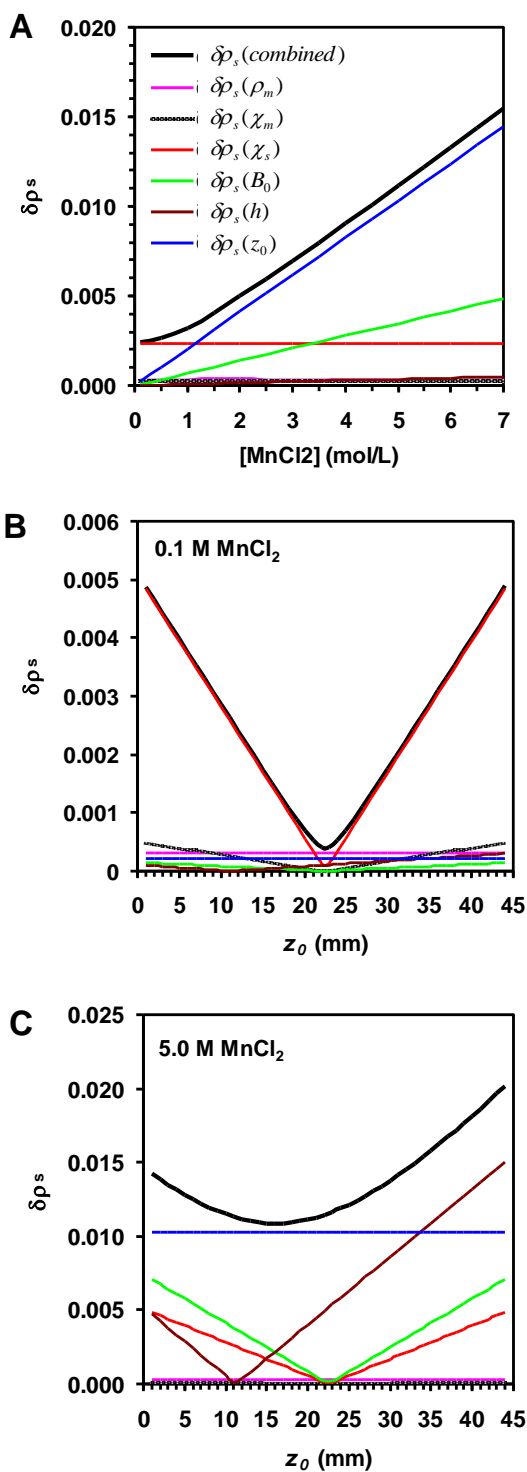
Table S4. Details of experimental parameters used for obtaining densities of solids using eqn (8) in MnCl_2 . Table 2 compares the calculated densities from Tables S3-S5.

| Sample | $[\text{MnCl}_2]$ (mol/L) | Density of MnCl_2 solution (g/cm^3) | χ_m (unitless) | z_0 (mm) | Calculated ρ_s (g/cm^3) |
|--|------------------------------|--|------------------------|----------------|--|
| glass beads | | | | | |
| d = 1.0100 | 0.100 | 1.0081 | 18×10^{-6} | 16.7 ± 0.5 | 1.0099 ± 0.0001 |
| d = 1.1000 | 1.000 | 1.0994 | 183×10^{-6} | 22.0 ± 0.5 | 1.101 ± 0.002 |
| d = 1.1500 | 1.500 | 1.1486 | 275×10^{-6} | 21.9 ± 0.5 | 1.152 ± 0.003 |
| spherical polymers | | | | | |
| polystyrene | 0.500 | 1.0492 | 92×10^{-6} | 23.5 ± 0.5 | 1.047 ± 0.001 |
| nylon 6/6 | 1.000 | 1.0994 | 183×10^{-6} | 13.5 ± 0.5 | 1.137 ± 0.002 |
| polymethylmethacrylate | 2.000 | 1.1971 | 367×10^{-6} | 23.3 ± 0.5 | 1.186 ± 0.004 |
| irregularly-shaped polymers | | | | | |
| polystyrene | 0.500 | 1.0492 | 92×10^{-6} | 22.0 ± 0.5 | 1.049 ± 0.001 |
| poly(styrene- <i>co</i> -acrylonitrile) | 1.000 | 1.0994 | 183×10^{-6} | 28.1 ± 0.5 | 1.076 ± 0.002 |
| poly(styrene- <i>co</i> -methylmethacrylate) | 1.000 | 1.0994 | 183×10^{-6} | 14.0 ± 0.5 | 1.133 ± 0.002 |
| organic droplets | | | | | |
| chlorobenzene | 1.000 | 1.0994 | 183×10^{-6} | 18.5 ± 0.5 | 1.115 ± 0.003 |
| 2-nitrotoluene | 1.000 | 1.0994 | 183×10^{-6} | 5.2 ± 0.5 | 1.170 ± 0.005 |
| dichloromethane | 3.000 | 1.2923 | 550×10^{-6} | 19.5 ± 0.5 | 1.329 ± 0.007 |
| 3-bromotoluene | 3.000 | 1.2923 | 550×10^{-6} | 12.5 ± 0.5 | 1.416 ± 0.007 |
| chloroform | 3.000 | 1.2923 | 550×10^{-6} | 6.5 ± 0.5 | 1.490 ± 0.008 |

Table S5. Details of experimental parameters used for obtaining densities of solids using eqn (8) in GdCl_3 . Table 2 compares the calculated densities from Tables S3-S5.

| Sample | $[\text{GdCl}_3]$ (mol/L) | Density of GdCl_3 solution (g/cm^3) | χ_m (unitless) | z_0 (mm) | Calculated ρ_s (g/cm^3) |
|--|------------------------------|--|------------------------|----------------|--|
| glass beads | | | | | |
| d = 1.0100 | 0.050 | 1.0100 | 18×10^{-6} | 26.0 ± 0.5 | 1.009 ± 0.001 |
| d = 1.1000 | 0.400 | 1.0950 | 140×10^{-6} | 20.2 ± 0.5 | 1.101 ± 0.002 |
| d = 1.1500 | 0.600 | 1.1426 | 210×10^{-6} | 20.5 ± 0.5 | 1.151 ± 0.003 |
| spherical polymers | | | | | |
| polystyrene | 0.200 | 1.0468 | 70×10^{-6} | 22.8 ± 0.5 | 1.046 ± 0.001 |
| nylon 6/6 | 0.400 | 1.0950 | 140×10^{-6} | 8.6 ± 0.5 | 1.133 ± 0.004 |
| polymethylmethacrylate | 0.750 | 1.1780 | 263×10^{-6} | 21.3 ± 0.5 | 1.186 ± 0.003 |
| irregularly-shaped polymers | | | | | |
| polystyrene | 0.140 | 1.0321 | 49×10^{-6} | 8.5 ± 0.5 | 1.050 ± 0.003 |
| poly(styrene- <i>co</i> - acrylonitrile) | 0.250 | 1.0589 | 88×10^{-6} | 12.7 ± 0.5 | 1.081 ± 0.002 |
| poly(styrene- <i>co</i> - methylmethacrylate) | 0.530 | 1.1260 | 186×10^{-6} | 20.9 ± 0.5 | 1.134 ± 0.003 |
| organic droplets | | | | | |
| chlorobenzene | 0.500 | 1.1189 | 175×10^{-6} | 27.7 ± 0.5 | 1.096 ± 0.003 |
| 2-nitrotoluene | 0.500 | 1.1189 | 175×10^{-6} | 12.2 ± 0.5 | 1.165 ± 0.003 |
| dichloromethane | 1.500 | 1.3514 | 526×10^{-6} | 26.2 ± 0.5 | 1.301 ± 0.007 |
| 3-bromotoluene | 1.500 | 1.3514 | 526×10^{-6} | 17.8 ± 0.5 | 1.415 ± 0.006 |
| chloroform | 1.500 | 1.3514 | 526×10^{-6} | 11.8 ± 0.5 | 1.496 ± 0.007 |

Figure S6. Plots showing the dependence of error in ρ_s on various experimental parameters at constant temperature. A) Dependence of $\delta\rho_s$ on $[\text{MnCl}_2]$ in water at 23 °C. Dependence of $\delta\rho_s$ on z_0 over the entire vertical distance between the magnets (45mm) at B) 0.1 M MnCl_2 and C) 5.0 M MnCl_2 at 23 °C.



References:

- (1) Lide, D. R., Ed. *CRC Handbook of Chemistry and Physics*, 89th ed. [Online]; CRC Press: Boca Raton, FL, 2008.
- (2) Du Tremolet de Lacheisserie, E., Gignoux, D., Schlenker, M., Eds. *Magnetism*; Kluwer Academic Publishers: Norwell, MA, 2002.
- (3) Hirota, N.; Kurashige, M.; Iwasaka, M.; Ikehata, M.; Uetake, H.; Takayama, T.; Nakamura, H.; Ikezoe, Y.; Ueno, S.; Kitazawa, K. *Physica B* **2004**, *346*, 267-271.
- (4) Ikezoe, Y.; Kaihatsu, T.; Sakae, S.; Uetake, H.; Hirota, N.; Kitazawa, K. *Energy Conv. Manag.* **2002**, *43*, 417-425.
- (5) Kang, J. H.; Choi, S.; Lee, W.; Park, J. K. *J. Am. Chem. Soc.* **2008**, *130*, 396-397.
- (6) Kuchel, P. W.; Chapman, B. E.; Bubb, W. A.; Hansen, P. E.; Durrant, C. J.; Hertzberg, M. P. *Concepts in Magn. Reson.* **2003**, *18A*, 56-71.
- (7) Simon, M. D.; Geim, A. K. *J. Appl. Phys.* **2000**, *87*, 6200-6204.
- (8) Taylor, J. R. *An Introduction to Error Analysis*; University Science Books: Sausalito, CA, 1997.
- (9) Andres, U. *Magnetohydrodynamic & Magnetohydrostatic Methods of Mineral Separation*; John Wiley & Sons: New York, NY, 1976.
- (10) Hatscher, S.; Schilder, H.; Lueken, H.; Umland, W. *Pure Appl. Chem.* **2005**, *77*, 497-511.
- (11) Weiss, R. F. *Deep-Sea Research* **1970**, *17*, 721-735.
- (12) Sohnel, O.; Novotny, P. *Densities of Aqueous Solutions of Inorganic Substances*; Elsevier: New York, NY, 1985.
- (13) Paramagnetic samples will also levitate in this device provided that $\chi_m > \chi_s$.