Supporting Information

Negative-Pressure Soft Linear Actuator with a Mechanical Advantage

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Fabrication of shear-VAMs

The elastomeric parts of shear-VAMs were created by replica molding (Figure S1). We designed the molds using computer-aided design (CAD) (Solidworks) and fabricated them with acrylonitrile butadiene styrene (ABS) plastic using a 3D printer (StrataSys Fortus 250mc). Curing a silicone-based elastomer (Ecoflex 00-30, dragon skin 10 slow, or Elastosil M4601) against the molds at room temperature (4 hours for Ecoflex 00-30, and 6 hours for dragon skin 10 slow and Elastosil M4601) produced two halves of the shear-VAMs. These two halves were aligned and bonded together by applying uncured elastomer at their interface, prior to curing once again at room temperature for the same amount of time.

The inextensible strips were fabricated by placing pieces of nylon mesh on top of an acrylic plastic board, then pouring the corresponding elastomer over the mesh. A wooden stick was used to smooth the top surface of the elastomer. The composite strip is then formed after curing the elastomer in a 60 °C oven for 15 min. Two holes 6-mm in diameter are punched at the end of each strip for fixation purpose. Another smaller 3.5-mm diameter hole is punched on one of the two strips on a shear-VAM so that tubing can go through and transduce pneumatic pressure. The body of shear-VAM and the two strips are assembled by applying the same elastomer that they are made of as glue, and curing them at room temperature for 6 hours.

A conically shaped piece of the elastomer was bonded to the side of the actuator to provide additional material that allowed tubing (Intramedic polyethylene tubing, ID 0.76mm) to be securely attached to the structure. The conical piece was first pierced by a cannula. The tubing was fed through the cannula, which was then removed to leave the tubing embedded in the shear-VAM. The tubing was secured by elastic deformation of the elastomer, which, as the tubing displaced some of its volume, reacted by applying pressure to close the hole surrounding the tubing.

Why the Young's modulus E of the elastomeric material used to fabricate a shear-VAM is proportional to is maximum force of actuation F_{max}

A similar analysis was described in our paper on linear-VAMs;[1] we have adapted the analysis here. Consider the body of the shear-VAM. Since the inextensible strips are not under substantial deformation during actuation, the two side-surfaces of the shear-VAM that touch the inextensible strip can be considered to be each in a fixed boundary condition relative to itself. The two surfaces are, however, not fixed relative to each other. Due to the C2 symmetry of the shear-VAM, the two surfaces must move parallel to each other during actuation, and a shear stress is applied during this actuation. Let τ be the shear stress due to the hanging weight—that is, the load F divided by the lateral-area of the undeformed actuator A. And let s be the shear strain between the two surfaces—that is, the relative movement of the two strips Δh divided by the length of the actuator L (Equation S1). We regard an actuator as a thermodynamic system of two independent variables that can independently change the state of the actuator—the difference of pressure ΔP and the loading stress τ. To a good approximation, the elastomer is incompressible. Hence the state of the actuator depends on difference of pressure ΔP, but not on the absolute pressures inside and outside the actuator. Since the experiment is insensitive to small change in temperature, we do not list temperature as a variable. Using the neo-Hookean model, we may obtain Equation S2 on the basis of dimensional considerations:

$$s = \Delta h/L \tag{S1}$$

$$s = g(\Delta P/E, \tau/E), \tag{S2}$$

where g is a function of two variables and E is the Young's modulus of the elastomeric material. If we make two actuators with indistinguishable geometric features, but of materials with different Young's moduli, we can plot s as a function of $\Delta P/E$ and τ/E. The two surfaces will fall on top of each other.

Notice that according to Equation S2, if one increases E, ΔP, and τ by a common factor k, one obtains the same strain s. Assuming E, ΔP, and τ are increased to $E' = kE$, $\Delta P' = k\Delta P$, and $\tau' = k\tau$, we have relationship S3, where if Equation S2 describes a shear-VAM made of an elastomer of modulus E then Equation S4 must describe a shear-VAM of indistinguishable geometry, but made of an elastomer of modulus E'.

$$\tau/E = \tau'/E' \tag{S3}$$

$$s = g(\Delta P'/E', \tau'/E'), \tag{S4}$$

We note that a shear-VAM of modulus E can lift weight $F = \tau A$ only if the absolute value of $s(\Delta P)$ monotonically increases with ΔP; note also that Equation S2 and S4 as a function of ΔP must be simultaneously monotonically increasing, if one of them is so. Therefore a shear-VAM of modulus E can lift weight $F = \tau A$ if and only if a shear-VAM of modulus E' can lift weight $F' = \tau' A$. In other words, the Young's modulus E of the elastomeric material used to fabricate a shear-VAM is proportional to the maximum load it can lift.

Measurement of thermodynamic efficiency

We generated the pressure-volume hysteresis curves by pumping water (an incompressible fluid) in and out of the shear-VAMs. The actuator was fully submerged in a 1-gallon container of water. The hydraulic actuation, and measurement of volume was performed with a syringe pump (Harvard Apparatus, PHD 2000), and the pressure measurement was performed with a pressure sensor (Transducers Direct, TDH31) connected to the syringe pump and the pressure transfer line (Figure S5). We fixated the actuator in a position that was submerged fully in water. We filled the actuators with water by submerging them in the container of water and deflating them several times until bubbles no longer emerged.

Within each test, we switched from deflation to inflation when the actuator had achieved approximately complete contraction (about 3 mL change in volume). We chose the rate of deflation and inflation to be 1 mL/min, which was sufficiently slow to achieve quasistatic conditions. We repeated the deflation-inflation cycle seven times.

The fluid used for inflation/deflation (water) is effectively incompressible that we could equate the volume decrease/increase of fluid in the syringe to that of the increase/decrease in the volume of the channels in the shear-VAMs. The shear-VAMs required removal of $V_0 = 3$ mL of water to achieve an actuation distance of $\Delta h = {\sim}4.7\ mm$, while lifting a 200 g test weight, and while the applied differential pressure ramped up from 0 kPa to 15 kPa. We calculated the thermodynamic efficiency η by dividing "energy out" E_{out} by "energy in" E_{in} (Equation S5). E_{out} was obtained by calculating the potential energy gain of lifting the weight ($m = 200$ g was the weight we used, and g is the acceleration due to gravity) (Equation S6). E_{in} was obtained by integrating the differential pressure with respect to the change in volume (Equation S7). This value is represented by the area under the P-V curve (Figure S5B).

$$\eta = E_{\text{out}}/E_{\text{in}}. \tag{S5}$$

$$E_{\text{out}} = mg\Delta h. \tag{S6}$$

$$E_{\text{in}} = \int_0^{V_0} P(V)\, dV. \tag{S7}$$

Over a total of six runs, we obtained an efficiency of $\eta = 35\% \pm 1\%$. Note that the loss of energy due to hysteresis was small compared to the work done by the syringe pump during actuation (Figure S5B). The loss of efficiency was mainly due to the storage of elastic energy in the elastomer, with a small contribution from hysteresis.

Approximate theoretical derivation of the force of actuation *F*

Due to conservation of energy, the generation of a higher force of actuation F of shear-VAM (or of any other pneumatic actuators) requires the supply of a higher difference of pressure ΔP. Although ΔP is limited to 1 atm under atmospheric pressure, we will show that the force of actuation F of shear-VAMs increases linearly to the "lateral-area" of the strip A under fixed ΔP, limited only by the tensile strength of the strips. We will show that, unique to shear-VAM, the effective cross-sectional area of a shear-VAM does not necessarily increase as we increase A, allowing the shear-VAM to generate a mechanical advantage.

We can derive (approximately) the force of actuation F a shear-VAM produces for a given difference of pressure ΔP (that is less than the critical difference of pressure of a shear-VAM ΔP_{crit}) through an analysis of virtual work.[2] When a difference of pressure ΔP is applied, the volume of the void chambers decreases, and the two strips move towards each other. Assume an infinitesimal reduction of angle α to $\alpha - \delta\alpha$ (in radian), while the "lateral-area" A moves under force $\Delta P \times A$, and the shear-VAM reduces its volume from $A \times \mathrm{a} \times \sin(\alpha)$ to $A \times \mathrm{a} \times \sin(\alpha - \delta\alpha)$. The pneumatics virtual work (pressure-volume work) done to the system is:

$$\delta W_{in} = \Delta P \times \delta V = \Delta P \times A \times (\mathrm{a} \times \sin(\alpha) - \mathrm{a} \times \sin(\alpha - \delta\alpha))$$

$$= \Delta P \times A \times \mathrm{a} \times \cos(\alpha) \times \delta\alpha \qquad \text{(S8)}$$

where δV is the change of overall volume of the elastomeric part of shear-VAM. Since the elastomer is incompressible, δV is also equal to the volume of air pumped out of the void chambers of the shear-VAM. In the meantime, the shear-VAM exerts force F over a distance of $\delta h = (a \times \cos(\alpha - \delta\alpha) - a \times \cos(\alpha))$. The virtual work output (force-distance work) by the system to lift the load is:

$$\delta W_{out} = F \times \delta h = F \times (a \times \cos(\alpha - \delta\alpha) - a \times \cos(\alpha))$$

$$= F \times \mathrm{a} \times \sin(\alpha) \times \delta\alpha \qquad \text{(S9)}$$

Dividing Equation S9 by Equation S8, we obtain:

$$\eta(\alpha) = \delta W_{out}/\delta W_{in} = F \times \tan(\alpha)\,/(\Delta P \times A) \qquad \text{(S10)}$$

where $\eta(\alpha)$ is the efficiency of the shear-VAM near angle α, defined as the ratio between work out δW_{out} and work in δW_{in} (Equation S10). Since we know $\delta W_{in} = \delta W_{out} + \delta W_{lost}$, where δW_{lost} is the elastic and inelastic energy lost in compressing the elastomers, we know $\eta(\alpha) < 1$. Equivalently, Equation S10 can be written as a formula for the force of actuation F:

$$F = \eta(\alpha) \times \Delta P \times A/\tan(\alpha) \qquad \text{(S11)}$$

The efficiency $\eta(\alpha)$ comes primarily from the elastic loss in collapsing the chambers, and should be, in principle, constant (for the same angle α) as long as the beam length a and the beam spacing (10 mm in this design) are fixed. This efficiency $\eta(\alpha)$ is approximately equal to the thermodynamic efficiency of the shear-VAM η. Using the data plotted in Figure 3D, the efficiency $\eta(\alpha)$ in two shear-VAMs of different length $L = 62\ mm, 32\ mm$ can be calculated using Equation S10: the efficiencies are about the same for the two different lengths L, at ~40%

(at $\Delta P \approx 2\ kPa$, $\alpha \approx 45°$), and they are similar to the total thermodynamic efficiency of the shear-VAM (~35%).

In this derivation, we assumed the contribution of out-of-plane deformation of the inextensible strips to Δh is negligible—this assumption is especially true if the strips are made of materials with high stiffness. For more flexible strips, the Δh will be larger than our estimation due to tilting of the elastomeric body at un-actuated state (Figure 4B shows an example of this tilting), resulting in an under-estimation of δh. We also assume that the deformation of the elastomeric membranes is negligible. This assumption results in an over-estimation of change in volume δV as a function of angle α. Overall, these assumptions results in an overestimation of F, since:

$$F \times \delta h = \eta \times \Delta P \times \delta V \tag{S12}$$

This overestimation, however, doesn't change the qualitative behavior of F. Thus we can still use Equation S11 to study the scaling properties of F, which is the purpose of this derivation.

Comparison of shear-VAMs to conventional pneumatic/hydraulic systems

Mechanical advantage is often present in ordinary pneumatic/hydraulic systems. In a simple system with two cylinders (input cylinder with area A_0 and output cylinder with area A), the mechanical advantage $MA = A/A_0$. There are two key differences between an ordinary pneumatic system and shear-VAMs. i) The direction of force applied is *perpendicular* to the surface in the ordinary system, while it is *parallel* for shear-VAMs. This difference in direction implies that pressure-based mechanical advantage (Equation 6) can be achieved, which is not possible in the ordinary system. ii) The mechanical advantage for shear-VAMs, in terms of force, can be higher than that for the ordinary systems. Force generated by the ordinary system is

$\Delta P \times A$, whereas force generated by shear-VAMs is given by Equation 4. Thus, for the same area A, and pressure differential ΔP, the ratio of the force generated by shear-VAMs to the force generated by ordinary system is $\eta/\tan(\alpha)$. For the current design (with $\eta \approx 35\%$), shear-VAMs can achieve greater mechanical advantage than ordinary systems for $\alpha < 20°$.

Experimental procedure for measuring the relationship between the force of actuation *F* and the difference of pressure Δ*P*

Figure S6 shows a schematic diagram for the testing setup that measures relationship between the force of actuation F of shear-VAMs and the difference of pressure ΔP applied across the inside and outside of the shear-VAMs. The bottom end of each shear-VAM is tied to a weight placed on a scale to measure the force of actuation of the shear-VAM. The forces did not exceed the weight and thus, the weight did not lift off the scale. The scale together with the weight acts as a strain gauge to measure the force of actuation F of the shear-VAMs.

We generated the difference of pressure ΔP (i.e. a partial vacuum) by pumping air out of the actuator with a syringe connected to a syringe pump (Harvard Apparatus, PHD 2000). The pressure measurement was performed with a pressure sensor (Honeywell ASDX005D44R) connected to the syringe pump and the pressure transfer line (Figure S6). The voltage signal from the pressure sensor is received through a DAQ (NI USB-6210) and read with National Instruments™ LabVIEW. We slowly extracted air using the syringe pump at a rate of 2 mL/min and recorded the force readings at predetermined values of pressure. We repeated the deflation-inflation cycle seven times to obtain error bars for Figure 3D.

"Shear-VAMs" actuated with positive pressure

We can vary the design of a shear-VAM to generate an actuation stress by inflating the structure rather than deflating it. Figure S8 and Movie S8 show a variant of shear-VAM made of

Ecoflex (E = 43 kPa) with four beams that is driven by positive pressure to lift a 50 g-weight. This actuator is made based on a shear-VAM but the elastomeric body is glued in the opposite direction to the strips so that the angle between the beams and the strips $\alpha < 90°$ is now $\alpha' = 180° - \alpha > 90°$. This configuration generates a smaller distance of actuation Δh than a normal shear-VAM.

SI References

[1] D. Yang, M. S. Verma, J.-H. So, B. Mosadegh, B. Lee, F. Khashai, E. Lossner, Z. Suo, G. M. Whitesides, Adv. Mater. Technol. 2016, 1.

[2] C. L. Dym, I. H. Shames, Solid Mechanics: A Variational Approach, Springer, 1973.

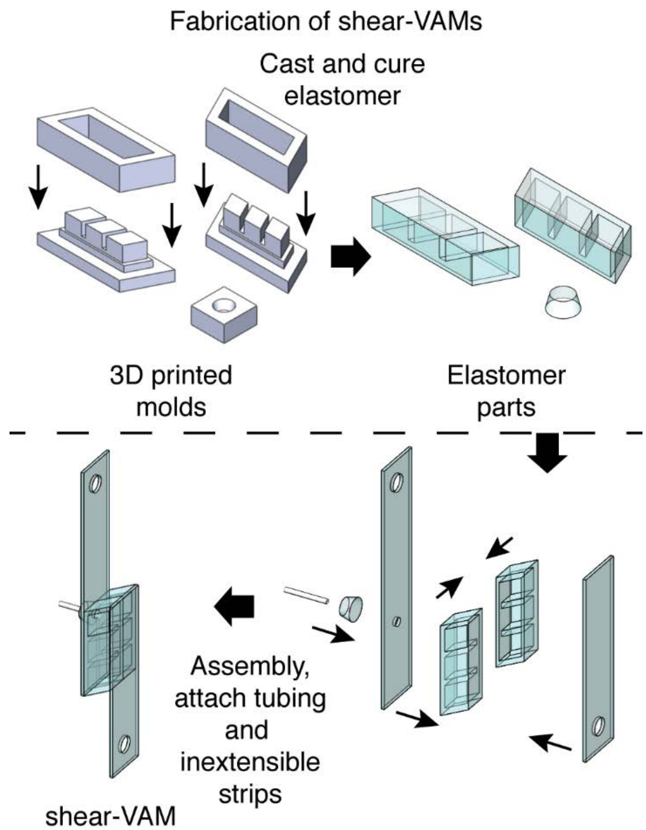

Figure S1. Fabrication of shear-VAMs.

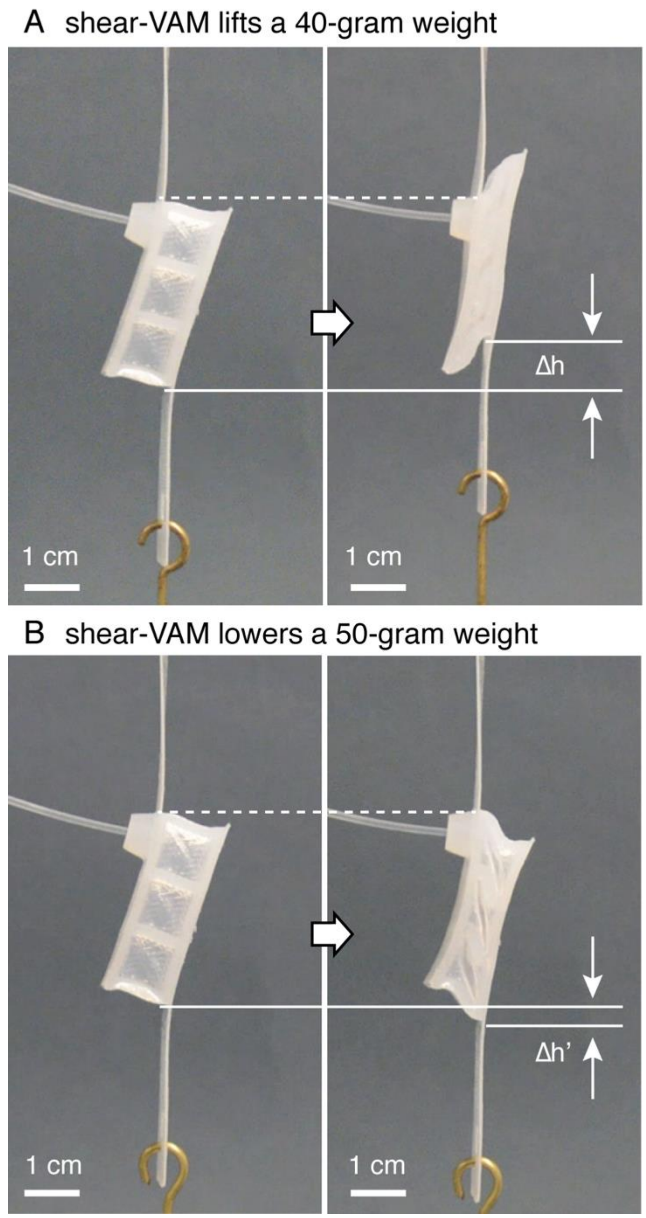

Figure S2. A shear-VAM contracts or extends on actuation depending on the load. A) The shear-VAM *lifts* the weight for a distance of Δh upon actuation while $F < F_{max}$. B) The shear-VAM *lowers* the weight for a distance of $\Delta h'$ upon actuation while $F > F_{max}$. The difference of pressure applied is ΔP = 90 kPa.

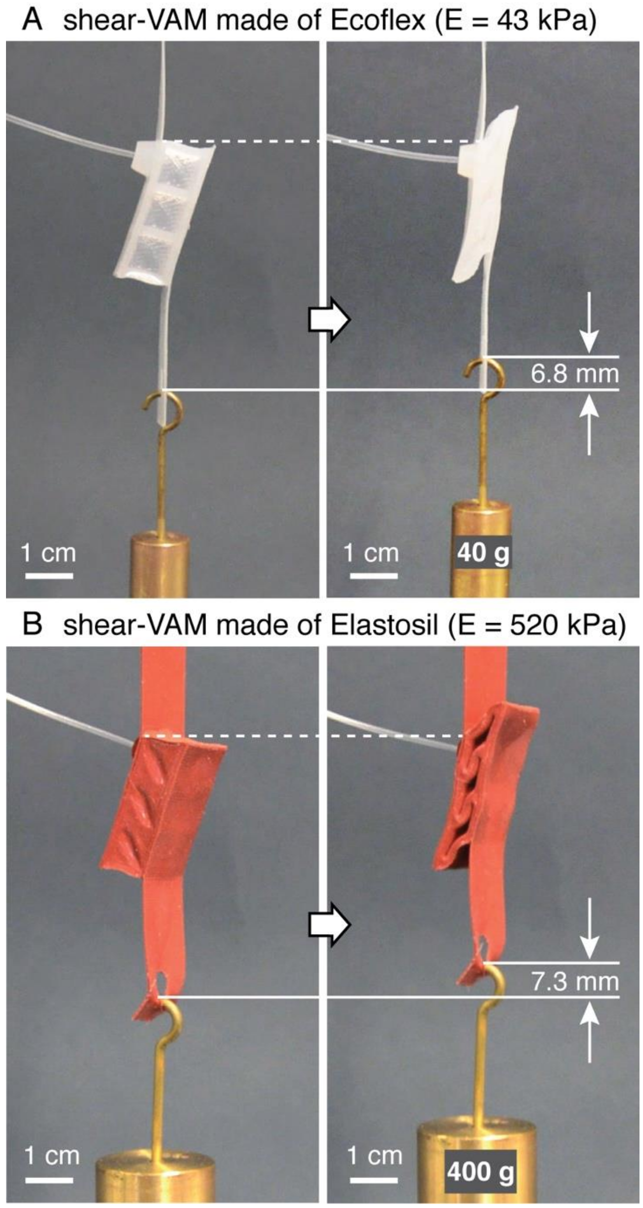

Figure S3. A shear-VAM fabricated from a stiffer elastomer can lift a heavier weight. A) A shear-VAM made of Ecoflex (E = 43 kPa) with four beams lifts a 40 g-weight (ΔP = 90 kPa). B) A shear-VAM of the same geometry but made of Elastosil (E = 520 kPa) lifts a 400 g-weight (ΔP = 90 kPa).

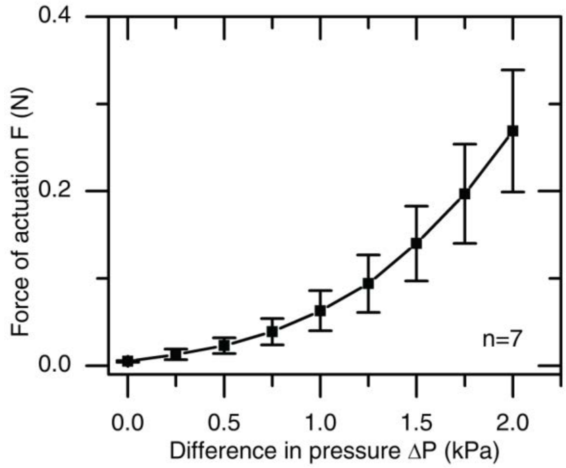

Figure S4. The force of actuation F of seven different shear-VAMs of length $L = 32\ mm$ (connected to a fixed strain gauge) vs. the difference of pressure ΔP (in kPa) applied across the inside and outside of the shear-VAMs.

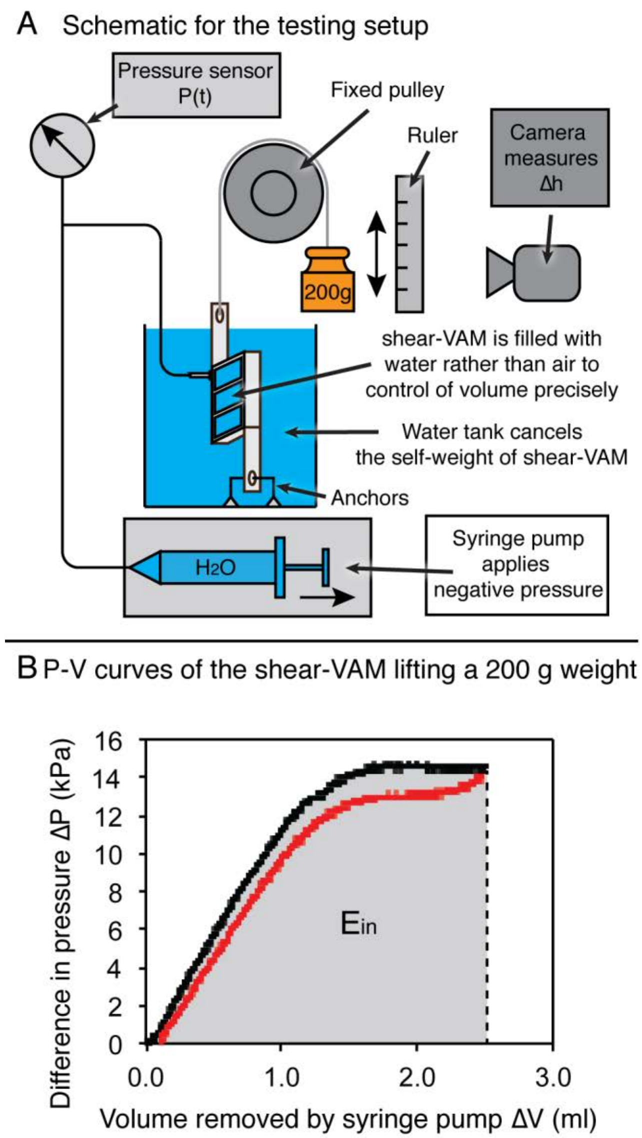

Figure S5. Experiment used to determine the thermodynamic efficiency of operation of a shear-VAM. (**A**) Schematic describing the setup used for testing. (**B**) P-V curves of a shear-VAM fabricated in Elastosil lifting a 500 g weight. The actuation curve is marked in black, and the return curve is marked in red. The shaded area $\boldsymbol{E}_{\mathbf{in}}$ represents the fluidic energy input via the syringe pump.

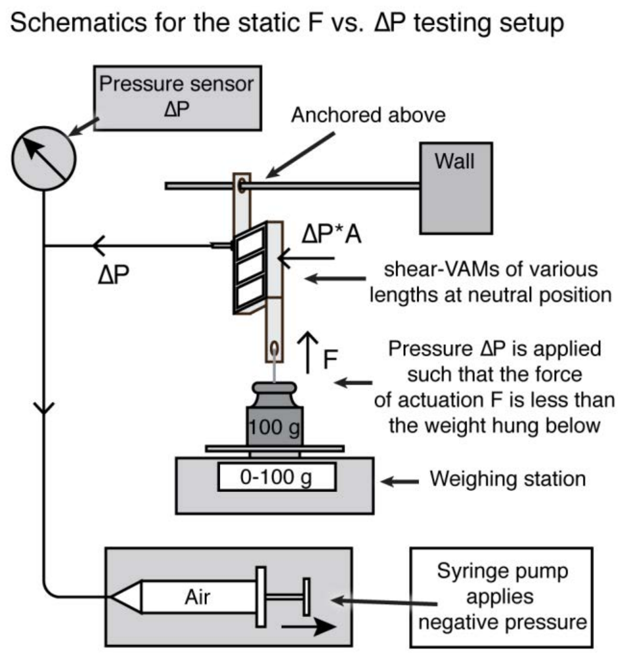

Figure S6. Schematic diagram for the testing setup that measures relationship between the force of actuation F of shear-VAMs and the difference of pressure ΔP applied across the inside and outside of the shear-VAMs. The bottom end of the shear-VAM is tied to a weight placed on a scale to measure the force of actuation of the shear-VAM. The forces did not exceed the weight and thus, the weight did not lift off the scale. The scale together with the weight acts as a strain gauge to measure the force of actuation F of the shear-VAMs.

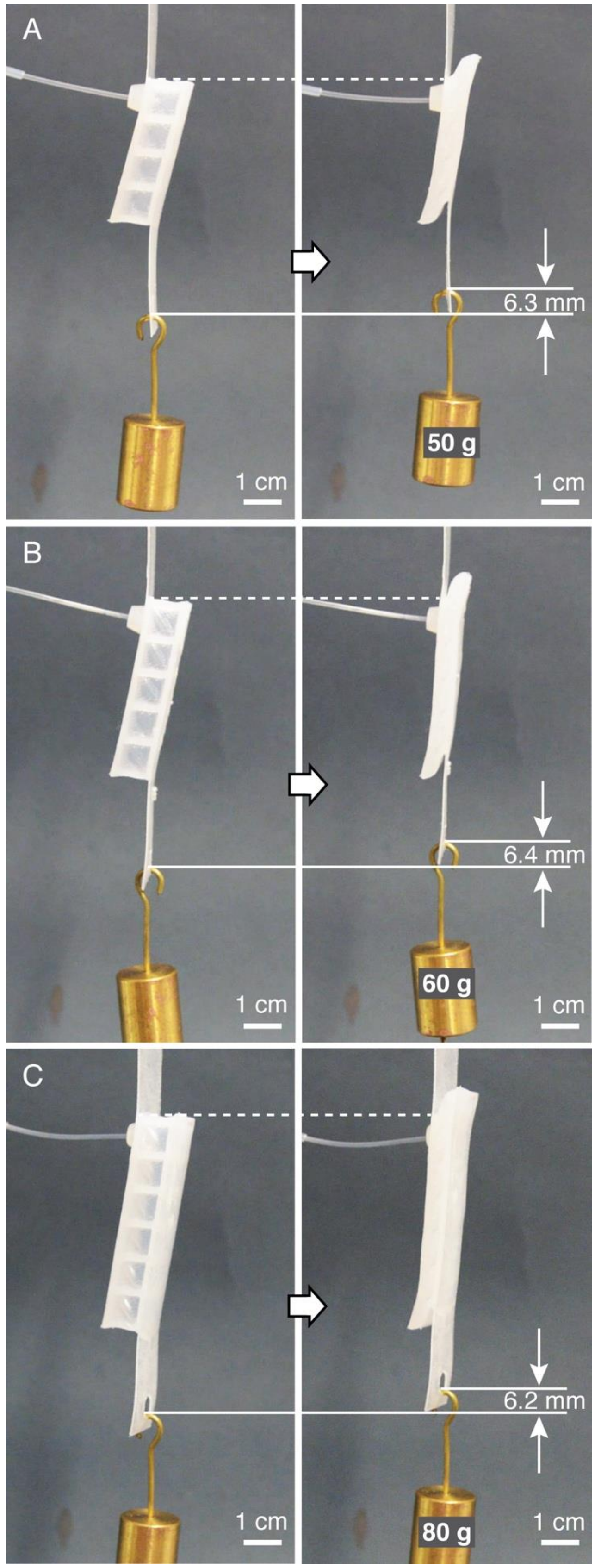
A
6.3 mm
50 g
1 cm
1 cm
B
6.4 mm
60 g
1 cm
1 cm
C
6.2 mm
80 g
1 cm
1 cm

Figure S7. The maximum force of actuation of a shear-VAM F_{max} (in Newtons) increases with its length L. A series of shear-VAMs of different lengths is tested to lift an increasing series of weights in 10 g-intervals (i.e. 10 g, 20 g, 30 g, etc.). The figure shows each of them lifting their highest weight in the series. A) A shear-VAM made of Ecoflex (E = 43 kPa) with five beams and length $L = 42\ mm$ lifts a 50 g-weight. B) A shear-VAM made of the same material but with six beams and length $L = 52\ mm$ lifts a 60 g-weight. C) A shear-VAM of the made of the same material but with seven beams and length $L = 62\ mm$ lifts an 80 g-weight.

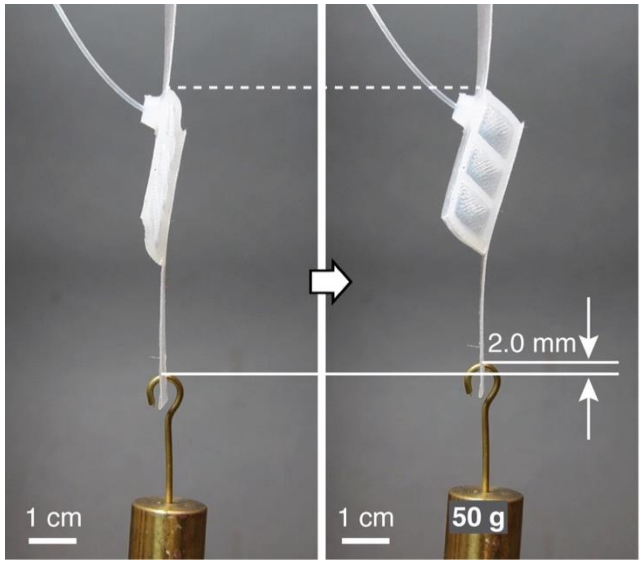

Figure S8. A variant of shear-VAM made of Ecoflex (E = 43 kPa) with four beams and length $L = 32\ mm$ that is driven by positive pressure lifts a 50-g weight. The distance of actuation Δh is less than that of a normal shear-VAM operated by vacuum.

Supporting Movie Legends

Movie S1. A shear-VAM fabricated in Ecoflex (E = 43 kPa) performing an actuation. The inside of the membrane is painted with a black marker to reveal the shape of the chamber more clearly. A black outline is visible after the actuator contracts.

Movie S2. When a shear-VAM is actuated, it could either perform a contraction or elongation depending on the load.

Movie S3. A shear-VAM fabricated in Ecoflex (E = 43 kPa) lifting a 40-g weight.

Movie S4. A shear-VAM fabricated in Elastosil (E = 520 kPa) lifting a 400-g weight.

Movie S5. Two shear-VAMs fabricated in Ecoflex (E = 43 kPa) in a parallel configuration lifting about twice as much as one such shear-VAM, but the same distance.

Movie S6. Two shear-VAMs fabricated in Ecoflex (E = 43 kPa) in series configuration lifting about the same weight as one such shear-VAM, but about twice the distance.

Movie S7. Two shear-VAMs in an agonist-antagonist configuration are able to drive a paddle used for a toy boat.

Movie S8. A paddle driven by two shear-VAMs in an agonist-antagonist configuration is able to move a toy boat in a water tank.

Movie S9. A shear-VAM where the beams are tilted in the opposite direction can be actuated with positive pressure.