Solving Mazes Using Microfluidic Networks

Michael J. Fuerstman,† Pascal Deschatelets,† Ravi Kane,† Alexander Schwartz,† Paul J. A. Kenis,† John M. Deutch,‡ and George M. Whitesides*,†

Department of Chemistry and Chemical Biology, Harvard University, 12 Oxford Street, Cambridge, Massachusetts 02138, and Department of Chemistry, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, Massachusetts 02139

Received February 12, 2003

This work demonstrates that pressure-driven flow in a microfluidic network can solve mazelike problems by exploring all possible solutions in a parallel fashion. Microfluidic networks can be fabricated easily by soft lithography and rapid prototyping. To find the best path between the inlet and the outlet of these networks, the channels are filled with a fluid, and the path of a second, dyed fluid moving under pressure-driven flow is traced from the inlet to the outlet. Varying the viscosities of these fluids allows the behavior of the system to be tailored. For example, filling the channels with immiscible fluids of different viscosities enhances the resolution of paths of different fluidic resistances.

Introduction

This paper demonstrates that pressure-driven flow in microfluidic systems can be used to solve the problem of finding the solution to a maze and of tracing the best path through the maze when there are several solutions. Conventional digital computation can solve simple mazes, where each path is assigned a weighting function, in an efficient manner;[1] more complicated systems that incorporate nonlinear behavior cannot be solved efficiently, if at all, by a computational algorithm. Three analogue methods have been used to solve mazelike problems: using the propagation of chemical waves,[2] utilizing tube morphogenesis in an amoeboid organism,[3] and tracing the path of a plasma.[4] Chemical waves propagate slowly, are difficult to visualize, and are inefficient at solving mazes with many solutions. Tube morphogenesis is very slow, is complex, and has been demonstrated only for very simple mazes. Plasma propagation is rapid and has a number of attractive features but only resolves paths through the maze based on their lengths.

We fabricate microfluidic mazelike networks in poly(dimethylsiloxane) (PDMS) and then inject colored tracer fluids into the mazes. The flow of fluid from the inlet to the outlet of the mazes determines which paths connect these points. The fluid will fill initially the best solution to the maze, where the "best solution" is defined as the path that offers the best compromise between channel width and channel length. We believe that this type of maze solving ultimately might be modified to take into account a range of complexities that will enable it to approximate real-world problems dealing with movement across large, complex systems. Such systems—e.g., distribution networks and highway systems—cannot be treated easily using conventional digital computation, because of their nonlinearities.[5]

We were attracted to using fluids to solve mazes, and to manipulate other extensive networks, because microchannels can be fabricated easily and because visualization of fluid flow is straightforward. This paper is a demonstration of principle in simple, but nontrivial, systems. The simplest mazes—those requiring only the determination of the shortest distance from entry to exit—can be solved in polynomial time by digital computational techniques.[6] Efficient algorithms also exist for solving networks of pipes, though these algorithms assume that the pipes are all cylindrical and that none of them bend.[7] Flows through channels with geometries as simple as square pipes with angular bends, the geometry present in our systems, can be modeled only with less accuracy and at greater computational cost than networks of channels of simpler geometries.[8] When the channels of the maze have complex properties—unidirectional flow, velocity-dependent resistance to movement, interaction between adjacent paths—the problem must be extended to involve detailed analysis of liquid flows; in those circumstances, digital computation is inefficient and can be inaccurate.

To explore solutions of a maze using microfluidics, we first fabricated the maze in the form of a network of microchannels. We normally then filled the maze with a fluid. Finally, a second fluid was introduced into this network and made to traverse it by applying a pressure gradient between the inlet and outlet. The fluid moves from the entrance of the maze (the inlet, maintained at a higher pressure) to the exit (the outlet, maintained at a lower one). In this type of system, the fluid explores all continuous paths simultaneously, and finds the best solution based on the path of least fluidic resistance across the network of microchannels. We can also modify various parameters of a microfluidic system—for example, the viscosity, velocity, and other parameters of the filling and displacing fluids—to modulate the behavior of the system.

* To whom correspondence should be addressed. Telephone: (617) 495-9430. Fax: (617) 495-9857. E-mail: gwhitesides@gmwgroup.harvard.edu.

† Harvard University.

‡ Massachusetts Institute of Technology.

(1) (a) Adamatzky, A. I. *Math. Comput. Modelling* **1996**, *23*, 105. (b) Dijkstra, E. N. *Numer. Math.* **1959**, *1*, 269.

(2) (a) Rambidi, N. G.; Yakovenchuck D. *BioSystems* **1999**, *51*, 67. (b) Agladze, K.; Magome, N.; Aliev, R.; Yamaguchi, T.; Yoshikawa, K. *Physica D* **1997**, *106*, 247. (c) Steinbock, O.; Tóth, Á.; Showalter, K. *Science* **1995**, *267*, 868.

(3) (a) Nagakaki, T.; Yamada, H.; Tóth, Á. *Biophys. Chem.* **2001**, *92*, 47. (b) Nagakaki, T.; Yamada, H.; Tóth, Á. *Nature* **2000** *407*, 470.

(4) Reyes, D. R.; Ghanem, M. M.; Whitesides, G. M.; Manz, A. *Lab Chip* **2002**, *2*, 113.

(5) (a) Sarmiento, A. M.; Nagi, R. *IIE Trans.* **1999**, *31*, 1061 b) Turner, S.; Nicholson, A. *Accid. Anal. Prevention* **1998**, *30*, 505.

(6) Cormen, T. H.; Leiserson, C. E.; Rivest, R. L. *Introduction to Algorithms*; The MIT Press: Cambridge, MA, 1990.

(7) Greyvenstein, G. P.; Laurie, D. P. *Int. J. Num. Methods Eng.* **1994**, *37*, 3685.

(8) Yuan, L. *J. Comput. Phys.* **2002**, *177*, 134.

10.1021/la030054x CCC: $25.00
Published on Web 04/29/2003

Background

Characteristics of Microfluidic Problem Solving. In this paper, we exploit the properties of pressure-driven flow in which one liquid displaces a second in a network of channels to solve the problems of finding the path of least fluidic resistance through a maze. We start by noting four characteristics of these systems. First, channels terminating in dead ends do not allow an appreciable incoming flow (with only a small amount of displacement resulting from eddies at the junctions of channels), since liquids are nearly incompressible. Second, in paths that proceed continuously without blockage from inlet to outlet, there is a pressure drop along the channel, and a flow in the network will occur along those paths. Third, the use of microfluidic systems (rather than macrofluidic systems) in our experiments prevents the occurrence of turbulence; fluid flows in microchannels are almost always at low Reynolds number (*Re*).[9] Laminar flow and the absence of turbulence are useful because they minimize mixing at branching points in the network and simplify the identification of the solution to the maze. Microfluidic systems are also smaller and easier to fabricate than macrofluidic ones. Fourth, injected liquid will move along all the paths that connect the inlet to the outlet (and that are, by definition, solutions of the maze). In the case of a system with more than one solution, tracing fluid will reach the outlet first along the path of the best solution (defined as the path of least fluidic resistance—that is, the best compromise between the shortest and the widest channels).

Background on Fluid Flow in Channels. Influence of the Geometry. Steady-state, pressure-driven fluid flow in a circular channel is described by the Hagen–Poiseuille equation, given as eq 1.[10] Q is the volumetric flow rate (m^3 s^{-1}), R is the radius of the channel (m), ΔP is the pressure difference between the inlet and the outlet (N m^{-2}), L is the length of the channel (m), and η is the dynamic viscosity of the fluid (N s m^{-2}). Volumetric flow rate can also be described by eq 2 where $\langle v_p \rangle$ is the average flow velocity (m s^{-1}). The combination of eqs 1 and 2 yields eq 3. For noncircular channels, R can be approximated by the hydrodynamic radius, R_H, which is defined as twice the cross-sectional area divided by the wetted perimeter.[11] In the case of a rectangular channel, R_H is defined in eq 4, where w and h are the width and height of the channel. By replacing R in eq 3 with the value of R_H given in eq 4, we obtain eq 5. Since the velocity of the flow is inversely proportional to the length of the channel, the amount of time, t, that the fluid takes to traverse the channel is proportional to the square of the length of the channel and inversely proportional to the square of the hydrodynamic radius. The formula is given in eq 6.

$$Q = \frac{R^4 \pi}{8L\eta} \Delta P \tag{1}$$

$$Q = \langle v_p \rangle \pi R^2 \tag{2}$$

$$\langle v_p \rangle = \frac{R^2}{8L\eta} \Delta P \tag{3}$$

$$R_H \equiv \frac{wh}{w+h} \tag{4}$$

$$\langle v_p \rangle \approx \frac{\Delta P}{8L\eta}\left(\frac{wh}{w+h}\right)^2 \tag{5}$$

$$t = \frac{8L^2\eta}{\Delta P}\left(\frac{w+h}{wh}\right)^2 \tag{6}$$

In the case of a microfluidic network in which multiple paths connect the inlet and the outlet, either ΔP is a constant for all paths between inlet and outlet or ΔP is 0 for a dead-end path unconnected to an outlet. For simple cases in which a single homogeneous, isotropic fluid is used, η is also a constant. For systems with these characteristics, eq 3 shows that the average velocity of the flow due to pressure along different solutions will depend on the width, height, and length of the path. More precisely, the average velocity will be inversely proportional to the length of the channel and directly proportional to the square of the hydrodynamic radius.

The Effect of Changing the Width of the Channels on the Average Velocity of a Fluid, When the Length and the Height of the Channels Remain Constant. The average velocity of the flow in a channel is proportional to R_H^2 (for a constant ΔP). The method of fabrication of our microfluidic mazes results in a constant height for all channels (the variation in channel height throughout the network is usually only 1–3%). Equation 5 shows that the average fluid flow velocity will be higher through wider channels than through narrower ones, for the same values of ΔP and L. We can look at how small changes in the width of the channel affect the flow velocity by taking $\partial\langle v_p \rangle/\partial w$ (keeping the height and length of the channel constant); the result of this derivative is given as eq 7.

$$\left(\frac{\partial\langle v_p \rangle}{\partial w}\right)_{L,h} = \frac{\Delta P}{4L\eta}\left(\frac{h^3 w}{(w+h)^3}\right) \tag{7}$$

The function in eq 7 reaches a maximum value when h/w is equal to 2.[12] Changing the width of the channel therefore produces the greatest change in $\langle v_p \rangle$ when the channel is approximately twice as high as it is wide. For $h/w \gg 2$ or $h/w \ll 2$, increasing the width of the channel has less of an effect on the average velocity of the flow than when $h/w \sim 2$. Systems with channels of different (and possibly varying) widths (such as the Boston street map described later) can use ratios of h/w around 2 to maximize the effect of variations of width on the velocity along the channels, while systems with channels of constant width could use h/w ratios significantly different from 2 to minimize the effect of differences in width and height due to defects or imperfections caused by the fabrication process.

The Effect of Changing the Length of the Channels on the Average Velocity of a Fluid, When the Width

(9) Kenis, P. J. A.; Ismagilov, R. F.; Whitesides, G. M. *Science* **1999**, *285*, 83.

(10) Landau, L. D.; Lifshitz, E. M. *Fluid Mechanics*, 2nd ed.; Course of Theoretical Physics; Pergamon: New York, 1987; Vol. 6.

(11) Delamarche, E.; Bernard, A.; Schmid, H.; Bietsch, A.; Michel, B.; Biebuyck, H. *J. Am. Chem. Soc.* **1998**, *120*, 500.

(12) We can derive this result by following the protocol for determining the maximum of a function. First, we take the derivative of eq 7 and solve it (that is, determine what values of h and w will make it equal to zero). Taking the derivative gives

$$\left(\frac{\partial^2\langle v_p \rangle}{\partial w^2}\right)_{L,h} = \frac{\Delta P}{4L\eta}\left(\frac{h^3(h-2w)}{(w+h)^4}\right)$$

which clearly equals zero either when $h = 2w$ or when w is infinitely large. To determine if $h/w = 2$ is a maximum of the function in eq 7, one must take the *third* derivative of $\langle v_p \rangle$ with respect to width, which gives

$$\left(\frac{\partial^3\langle v_p \rangle}{\partial w^3}\right)_{L,h} = \frac{3\Delta P}{2L\eta}\left(\frac{h^3(w-h)}{(w+h)^5}\right)$$

Plugging in $h = 2w$ to this equation gives us $-wh^3$ in the numerator of the fraction, which is always negative for positive values of width and height. This means that the function in eq 7 is concave-down at $h/w = 2$, and thus $h/w = 2$ is a maximum of the function in eq 7.

and the Height of the Channels Remain Constant. A fluid will move faster in shorter channels than in longer ones, because the pressure gradient along the channel is higher. Equation 5 correspondingly shows that $\langle v_p \rangle$, the average flow velocity of a fluid due to a pressure drop, is inversely proportional to the length of the channel. The time required for a given volume of fluid to reach the outlet is proportional to L^2, according to eq 6. Thus, an element of fluid in shorter channels will reach the outlet before a corresponding element in longer ones.

Analogy to Networks of Resistors. To a first approximation, fluidic networks can be compared to a network of ohmic resistors, by making the following substitutions:[13]

$$\text{current} \leftrightarrow Q$$

$$\text{voltage} \leftrightarrow \Delta P$$

$$\text{resistance} \leftrightarrow \frac{8L\eta}{\pi {R_H}^4}$$

Substitution of these values into Ohm's law ($V = IR$), where V is the voltage across a resistor of resistance R and I is the current passing through it, generates eq 5.

The overall fluidic resistance of a network obeys the same rules as networks of resistors. When resistances R_1 and R_2 are added in series, the total resistance, R_3, is simply the sum of R_1 and R_2. If the two resistors are added in parallel, the total resistance is found using eq 8.

$$R_3 = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} \tag{8}$$

We emphasize that this correspondence between fluid flow and current (flow of electrons) holds only for simple, Newtonian fluids. Complex fluids, such as non-Newtonian fluids (the viscosities of which change with the application of shear forces),[14] would require much more complex electrical analogies (if, indeed, such analogies existed at all). Additionally, it is sometimes difficult to obtain an expression for the fluidic resistance of a channel, even for Newtonian fluids.

Results and Discussion

Fabrication of the Mazes. The mazes were fabricated using soft lithography and rapid-prototyping techniques.[15−18] With the exception of the map of Boston, we created the layouts of all of the mazes ourselves. We note that the maze layouts were not generated randomly by a computer program; we planned each maze so that it would contain a predetermined number of solutions. Figure 1a shows a representative maze that was fabricated in poly(dimethylsiloxane) (PDMS) from a master made of SU-8 photoresist. The master was 50 μm thick and was composed of patterns of photoresist 100 μm wide. It was silanized prior to use.[19] We cured PDMS prepolymer on this master and sealed this imprinted PDMS piece against a clean, flat surface to close the channels;[15] a flat piece of PDMS, a glass slide, or a piece of silicon tape served equally well to seal the channels.

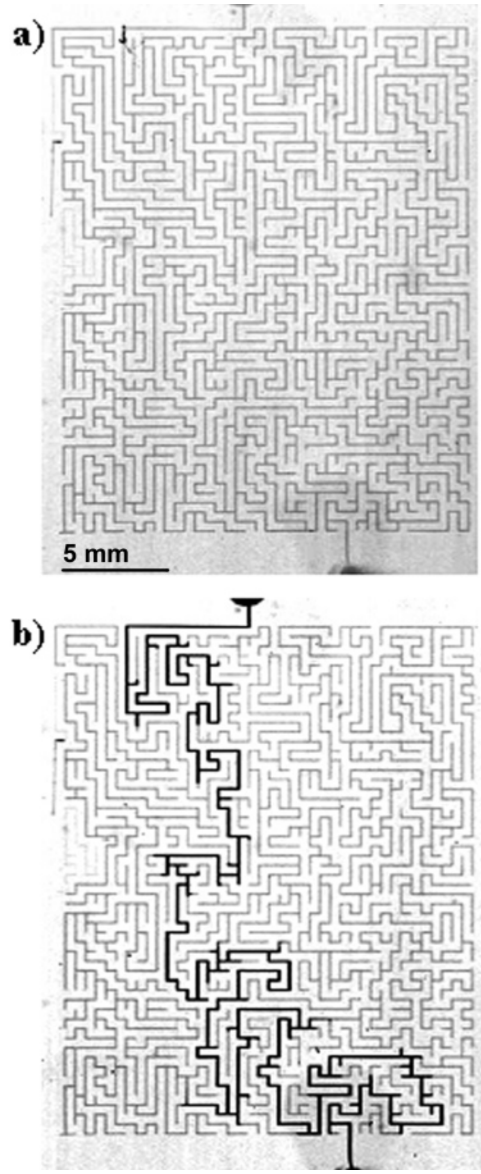

Figure 1. (a) A one-solution maze filled with a solution of bromophenol blue in ethanol. (b) The same maze after injecting a second solution of commercial food dye in a water/EG mixture into the inlet located at the top using a syringe pump. The purple stream found the way to the outlet at the bottom of the pictures.

The Choice of the Combination of Liquids and Dyes. The simplest method for testing the ability of microfluidic systems to solve mazes is to fill a maze (having channels with a constant cross section) with a liquid and then to inject a colored solution of the same liquid in order to visualize the paths connecting the inlet and the outlet of the maze. The viscosity is constant in the network, and there are no capillary forces anywhere inside of the network of channels. Unfortunately, the parabolic flow profile associated with laminar Poiseuille flow in microchannels severely broadens the edges of plugs of dye and prevents us from pinpointing the position of the flow in a microfluidic network comprising channels filled with a single liquid.

We examined a variety of fluids and dyes in order to find a combination that allowed us to visualize satisfactorily the position of the edge of a colored fluid flowing

(13) Kirshner, J. M.; Katz, S. *Design Theory of Fluidic Components*; Academic Press: New York, 1975.
(14) Barnes, H. A.; Hutton, J. F.; Walters, K. *An Introduction to Rheology*; Rheology Series; Elsevier: Amsterdam, 1989; Vol. 3.
(15) Duffy, D. C.; McDonald, J. C.; Schueller, O. J. A.; Whitesides, G. M. *Anal. Chem.* **1998**, *70*, 4974.
(16) Qin, D.; Xia, Y.; Whitesides, G. M. *Adv. Mater.* **1996**, *8*, 917.
(17) Xia, Y.; Whitesides, G. M. *Angew. Chem., Int. Ed. Engl.* **1998**, *37*, 550.
(18) McDonald, J. C.; Duffy, D. C.; Anderson, J. R.; Chiu, D. T.; Wu, H.; Schueller, O. J. A.; Whitesides, G. M. *Electrophoresis* **2000**, *21*, 27.
(19) Chaudhury, M. K.; Whitesides, G. M. *Langmuir* **1991**, *7*, 1013.

inside the network. We tested a variety of combinations of fluids using fluids that would not swell PDMS, in conjunction with different dyes. We examined perfluoro(methyldecalin) (PFMD), hexane, ethanol, water, and ethylene glycol (EG), along with aqueous and ethanol-based solutions of dyes such as bromophenol blue, phenol red, McCormick brand food coloring composed mainly of water and propylene glycol (McCormick, Hunt Valley, MD), and Waterman brand office ink (Waterman Pen Company, Janesville, WI). Any liquid selected to fill the system initially had to wet PDMS appreciably, as displacing air bubbles trapped in the system was otherwise an exercise in futility. Water filled the channels efficiently (as air in dead-end channels was forced to diffuse out of the microfluidic system through the PDMS) when the PDMS was freshly plasma oxidized. This filling process became much less efficient if the channels were not filled within a few hours of the time when they were oxidized, since the oxidized PDMS reorganized to a form with lower surface free energy (by mechanisms that have not been defined in detail).[20] By contrast with water, ethylene glycol and ethanol filled the channels without difficulty, regardless of the free energy of the surface.

These observations are valid for PDMS but will be different for microchannel systems made of other materials, since a change in surface energy reflecting a change in the material used to fabricate the microfluidic channel will modify the flow profile along the channel. The combination of miscible fluids and dye that gave the best qualitative results (i.e., the most sharply defined interface between dyed and undyed fluids) in our experiments was ethanol (as a filling fluid) and food dye in water and ethylene glycol (as a tracer fluid). In some experiments, we added glycerol to this colored solution in order to increase its viscosity. In a two-fluid experiment involving miscible fluids, we usually filled the network of channels initially with ethanol, and the second, more viscous liquid was then injected into the inlet. The dye did not diffuse very rapidly into ethanol, so the leading edge of the dye plug remained relatively sharp and easy to locate. We found that using immiscible fluids, particularly when one of the fluids was perfluorinated, further optimized the sharpness of the fluid interface in two-fluid experiments. For example, when we used a combination of PFMD as a filler fluid and Waterman pen ink as a tracer fluid, as in Figure 2, the interface between the two immiscible fluids remained extremely sharp during pressure-driven flow.

A microchannel system containing two different fluids is more complex than one containing a single fluid. The average flow velocity due to pressure, $\langle v_p \rangle$, when using a two-fluid system, cannot be described using a single value of η. In addition, the average velocity, $\langle v_p \rangle$, will also be influenced by the capillary force present at the interface between the two fluids. The relative importance of this capillary force can be limited by injecting fluids under high pressures, since the pressure-driven flow can be made to be much faster than the capillary-driven flow. Experimentally, high-viscosity solutions were sometimes used in order to keep the observed flow velocities reasonably low (e.g., the aqueous mixture of the McCormick food dye at 20 °C has a viscosity of ~11 cP; the viscosity of water is 1 cP). Using two liquids with different viscosities conserves the trends in the effects of the width and length of the channel on average flow velocity: the highest average fluid velocity was observed along the pathway offering the least fluidic resistance. The relative velocity

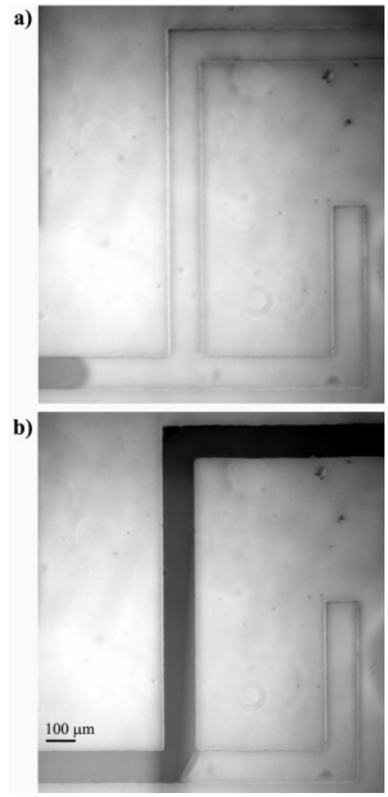

Figure 2. (a) Maze channels filled with PFMD (colorless). Ink displaced the PFMD from the left. The ink/PFMD interface was well defined and parabolic. Note that the path that continues to the right is a dead end, while the path that goes upward leads to the outlet. (b) The same channels after the ink had passed the intersection with the dead end. The PFMD did not flow into the dead-end channel.

between the flows through two different valid paths through the maze might change when fluids of different viscosities are used, but a one-fluid and a two-fluid system choose the same path as the "best" solution, provided that the flows in each channel rapidly assumed their steady-state values. A further discussion of the effect of using fluids of different viscosities is given in the section titled Experiments Using Two Fluids with Different Viscosities.

For every experiment described, injection of liquid was done at a constant flow rate using a syringe pump. The average flow velocity was adjusted to be in the range of 5−10 mm/s. The pressure difference between the inlet and the outlet was in the range of 100−300 Pa. These parameters resulted in the liquid flow taking 5−30 s to travel from the inlet to the outlet, for the microfluidic systems used.

A Maze with One Solution. We fabricated a maze with one open path from the inlet to the outlet. As this maze had only one possible valid solution, we refer to it as a "one-solution maze". Figure 1a shows a one-solution maze that was first filled with an ethanol-based solution of bromophenol blue whose concentration was qualitatively adjusted to produce a light coloration. A darker solution of McCormick food dye in a water/EG mixture was then injected. The dark stream in Figure 1b shows the shortest path between inlet and outlet.

In this experiment—and others in which miscible fluids were used—the darker solution advanced some distance into the dead-end channels and then stopped (Figure 1). When the flowing stream of water/EG encountered these side channels, eddies formed rapidly at the entrances. These eddies could be visualized readily using suspensions

(20) Morra, M.; Ochhiello, E.; Marola, R.; Garbassi, F.; Humphrey, P.; Johnson, D. *J. Colloid Interface Sci.* **1990**, *137*, 11.

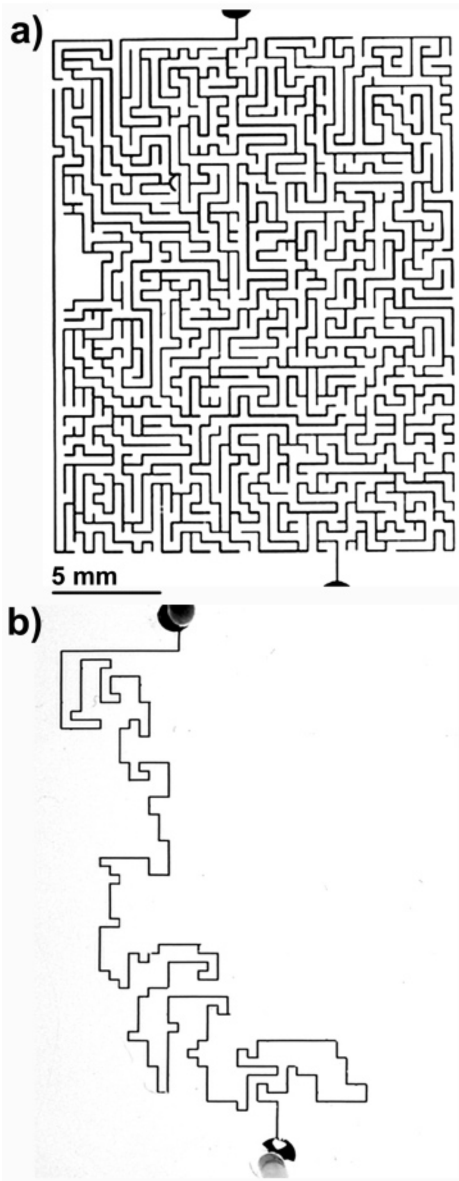

Figure 3. (a) A maze was filled with an aqueous ink solution. A small channel that is completely disconnected from the rest of the maze causes a region in the left-hand side of the maze to remain unfilled. That this section is not filled indicates adequate sealing between the PDMS maze and the glass slide that forms the floor of the channels. (b) The same maze was first filled with PFMD, with the ink then added as the tracer fluid. Again, we observed minimal intrusion of the ink into the dead-end channels. The ink took 31 s to traverse this network.

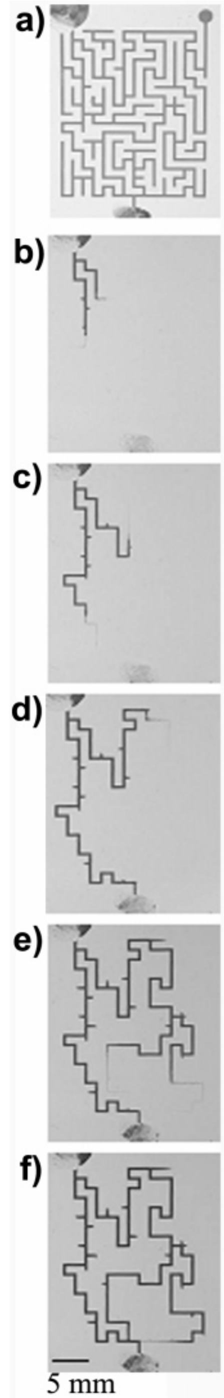

Figure 4. (a) A three-solution maze entirely filled with a solution of food dye in a water/EG mixture. (b−f) The same three-solution maze filled entirely with ethanol before the water/EG solution of commercial food dye was added from the top. The pictures were taken in 1.5 s intervals. (d) The dark stream found the shortest path to the outlet (bottom). The stream advanced more slowly in the channels associated with the two other solutions to the maze than it did in the channel that composes the shortest path.

of microbeads. Eddies mix the ethanol present in these entrance regions with the colored water/EG solution flowing along the paths leading to the outlet; in less than a second after the colored water/EG stream had contacted the ethanol, the water/EG solution completely replaced the ethanol in the region involved in these eddies. Using immiscible liquids (such as PFMD and an aqueous ink solution) practically eliminated such mixing (Figure 3).

The Best Path in a Maze with Three Solutions. This technique will also solve mazes in which the inlet and outlet are connected by more than one path. Figure 4 shows a maze that has three different paths connecting the inlet and the outlet (three solutions). The dark stream reached the outlet first by the pathway of least fluidic resistance. Since the widths of the channels are the same throughout the maze, this path is also the shortest of the possible maze solutions. The stream also advanced in the channels associated with longer solutions, but at a slower pace.

Experiments Using Two Fluids with Different Viscosities. Microfluidic systems offer the possibility of using fluids with a wide range of properties. This flexibility allows us to adjust certain behaviors of the system—for example, the relative velocity of fluids along different paths

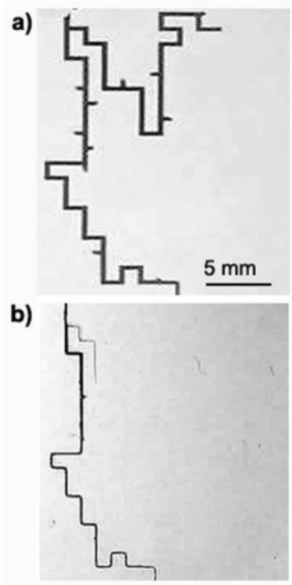

Figure 5. In these experiments, we took a picture the moment the tracer fluid first reached the outlet, located at the bottom of each picture. a) We injected an aqueous solution of food dye into a three-solution maze filled with air. b) We filled the same maze with a 1:4 mixture of glycerol and water ($\eta = 60$ cP) before injecting a solution of bromophenol blue in ethanol into the inlet.

in a system—without changing the geometry of the channels. We used two fluids with different viscosities to influence the difference in fluid velocity between the shortest path and the other paths in a multiple-solution maze (Figure 5). When we injected a fluid into a maze filled with air, the velocity of flow was largely independent of the channel length, because the pressure drop across the portion of the channels filled with air was negligible. When it reached the outlet along the shortest path, the fluid had traveled the same distance along the second, longer path (Figure 5a). The sensitivity of the microfluidic maze solver for the path of least hydrodynamic resistance can be increased by filling the microfluidic network with a fluid of high viscosity before injecting a colored tracer fluid of lower viscosity (Figure 5b). In this case, the average viscosity along a given path decreases as the tracer fluid progresses; this decrease results in an increase in the velocity of the interface between the tracer and the filling fluids. Since the tracer fluid is already moving faster along the shortest path than the longer ones, the *average* viscosity along this path decreases more quickly than along other solutions of the maze. The average viscosity of the fluid in the shortest channel decreases faster than the average viscosity in the longer channels; thus, there is a greater increase in velocity along the shortest path relative to the others. This effect therefore progressively favors the pathway of least hydrodynamic resistance as the displacing fluid proceeds.

One can also use the analogy of fluidic channels to a network of resistors to derive this behavior. Consider a channel of length L that is first filled with a fluid of some viscosity, η_2, and then is filled with a solution of a different viscosity, η_1. For example, Figure 6 shows a channel in which a dark, aqueous ink solution is displacing PFMD. If the ink occupies a segment of the channel of length L_1, and the PFMD occupies a segment of the channel of length L_2, then the resistance of each segment of the channel is $8\eta_1 L_1/R_H^2$ and $8\eta_2 L_2/R_H^2$, respectively. Thus, our effective resistance for this network of two serial resistors is equal to $8(\eta_1 L_1 + \eta_2 L_2)/R_H^2$. Using Ohm's law for the analogues

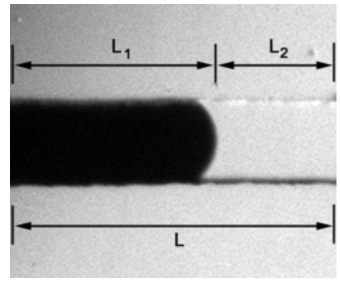

Figure 6. Black water-based ink displacing PFMD from a microchannel. The ink, which has a viscosity of η_1, occupies a length L_1; the PFMD, which has a viscosity of η_2, occupies a length L_2. Note that L is the sum of L_1 and L_2.

of current, voltage, and resistance given in the Analogy to Network of Resistors section, and substituting $\langle v_p \rangle$ as noted in eq 3, we obtain the expression in eq 9. If the second fluid added to the channel is less viscous than the first (that is, if η_1 is less than η_2), the average flow rate of the displacing fluid will increase as it fills the channel. If there are two channels being filled simultaneously, the average flow rate will increase more rapidly in the shorter channel than in the longer one, since the value of L will be smaller in the shorter channel than in the longer one.

$$\langle v_p(L_1) \rangle = \frac{R_H^{\,2}\Delta P}{8\eta_2}\left(\frac{1}{L - \left(1 - \frac{\eta_1}{\eta_2}\right)L_1}\right) \qquad (9)$$

We emphasize that eq 9 is only rigorously applicable to the situation in which one channel is being filled, since we add the resistances of the two segments of the channel in serial. If, as in these mazes, there are two channels connected in parallel, one must consider the resistances of both channels added in parallel. In the case of channels connected in parallel, however, the general conclusion of eq 9—that the average flow rate will increase more rapidly as the value of L decreases—will still hold.

Finding Optimal Routes in a Maze with a Large Number of Solutions. We used an urban street configuration as an example of a maze with a large number of different solutions. Figure 7 shows a maze fabricated by analogy to the street map of downtown Boston, but showing only a limited number of streets. We set the widths of the channels so that the ratios of velocities of fluid flow through the channels of different widths would match the ratios of typical driving speeds on the corresponding roads. Thus, minor streets were represented by channels 100 μm wide, major streets by channels 200 μm wide, Storrow Drive (a four-lane road) by a channel 250 μm wide, and major highways by channels 300 μm wide. The height of all channels was fixed at 55 μm. Average velocity ratios were expected to be 1:1.5:1.6:1.7, respectively, across these channels of varying width, for the same pressure drop. These velocity ratios were chosen as a crude approximation of relative rates of traffic movement along the different types of roads. We started with a maze filled with a 1:4 mixture of water/glycerol ($\eta = 60$ cP) and injected a solution of bromophenol blue in ethanol ($\eta = 1.2$ cP) from an inlet to mark a starting point (the Boston Science Museum); an outlet marked the ending point (the Computer Museum) (Figure 7b–e). The system revealed paths between these two points. The use of a high viscosity liquid to prefill the maze helped to increase the difference in velocity between liquid flowing along the fastest solution and the slower solutions and made it easier to identify the results. The microfluidic networks used in these experi-

Figure 7. (a) Microfluidic network based on a map of Boston completely filled with an aqueous food dye in a water/EG mixture. Only some of the main streets of the city are included, with major arteries and interstate highways represented by larger channels than the smaller roads. (b−e) A solution of bromophenol blue in ethanol flowed into the network that was prefilled with a 1:4 mixture of water and glycerol. (e) Two routes emerged, with one being slightly faster.

ments are not faithful models of the actual flow of traffic in an urban area, but the street map of Boston is a good example of a complex network of interconnecting pathways. This map illustrates the possibility of fabricating networks composed of multiple channels of different widths. We believe that this capability to modify resistance to flow is one of the useful characteristics of a fluidic maze solver (as opposed to computational or plasma-based maze solvers) and that it will eventually prove useful in building accurate models of complex systems.

Solving Mazes with Multiple Solutions of Similar Fluidic Resistances. To test the ability of these systems to resolve solutions of similar resistance, we fabricated a maze of channels of uniform width containing two possible solutions. The shortest path had a length of 84.5 mm; the longer path was 86.5 mm long. The difference in length of the two paths was thus 2.4%. Since both paths made the same number of right-angle turns from inlet to outlet, and since the heights and widths of the paths ideally were constant and equal, the shorter path had a lower resistance and hence was expected to fill first. The maze was initially filled with PFMD ($\eta \approx 5$ cP), with an aqueous solution of Waterman office ink ($\eta \approx 1$ cP) used as the tracer fluid. Figure 8 shows that the maze solver was able to distinguish between these two solutions.[21] Moreover, since the tracer fluid was significantly less viscous than the fluid it was displacing, the tracer advanced only 77 mm into the 86.5 mm channel in the time that the 84.5 mm channel filled completely (in these experiments, since the filler and tracer fluids were immiscible, intrusion of the ink into dead ends was minimal). Thus, a 2.4% difference in the lengths of the two channels resulted in a 9.9% difference in the distance the fluid traveled through the channels. This experiment was run several times with different replicas of this maze, each time resulting in a similar resolution of the two paths.

In reality, the channels in the maze did not have completely uniform height and width; the channels composing the longer solution had an average height of 170.9 μm and an average width of 91.9 μm, whereas the channels composing the shorter one had an average height of 169.7 μm and an average width of 91.2 μm. To determine, to a first-order approximation, how this discrepancy affects the amount of time each path through the maze takes to fill completely with ink, we use eq 6. Using the data above and eq 6, we calculate that the ratio of the time through the shorter path to the time through the longer path is 0.969.[22] This means that the ink travels through the longer path in a period of time that is 3.2% longer than the amount of time it takes to travel through the shorter path. If the longer path had the same height and width as the shorter path, the ratio of filling times would be dependent only on the lengths of the channels; the ink would take 4.8% longer to traverse the longer path than it would to go through the shorter one. It is therefore evident that, though the nonuniformity in the widths and heights of the channels in this maze affects the flow velocity to a degree, it does not change the qualitative result that the shorter path is favored over the longer one.[23]

When the maze was filled initially with ink and *then* infused with the more viscous PFMD, the resolution of the two paths was much poorer. The aqueous dye solution

(21) The images in Figures 3 and 8 were taken with a Sony DSC-F505V digital still camera. They were converted from color to gray scale, and their contrast and brightness adjusted using Adobe Photoshop to improve the visual discrimination between fluids. The images in Figures 1, 4, 5, and 7 were taken with a Nikon Coolpix 990 digital camera. The images in Figure 2 and Figure 6 were collected using an inverted microscope and a CCD still-image camera.

(22) Equation 6 does not take into account that the maze contains two fluids of different viscosities. As we have shown, using a less viscous fluid to push a more viscous fluid out of the channels favors the shorter path.

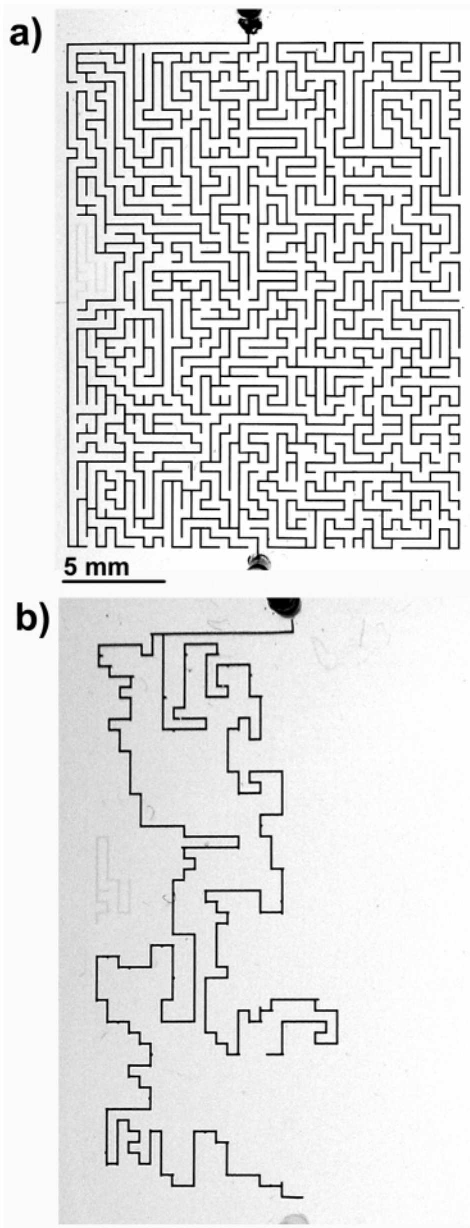

Figure 8. (a) A two-solution maze filled completely with an aqueous ink solution. (b) The same maze filled initially with PFMD, then injected subsequently with ink. The path that is filled completely with ink is 84.5 mm long, while the other path is 86.5 mm long. Due to the difference in viscosities of the two fluids used, the ink advanced only ~77 mm into the longer channel.

tended to adhere to the walls of the channel, preventing the PFMD from completely filling the channels through which it flowed. Moreover, since the less viscous ink is displaced by *more viscous* PFMD, we do not observe the increase in resolution of the two paths that we observed when PFMD was displaced by the aqueous ink solution. It is therefore evident that optimal resolution of paths of similar fluidic resistance is achieved if two immiscible fluids of significantly different viscosities, such as PFMD and aqueous ink solution, are used, with the less viscous fluid displacing the more viscous one.

(23) An analysis of eq 6 reveals that a necessary (but not sufficient) condition for the longer path to fill *before* the shorter one is that the ratio of the height of the longer channel to that of the shorter one, or the ratio of the width of the longer channel to that of the shorter one, must be greater than the ratio of the length of the longer channel to that of the shorter one.

Conclusion

This paper shows that microfluidic systems can be used to identify the pathways of least fluidic resistance through a maze. This method of solving a maze is massively parallel; it explores all pathways simultaneously. Although simple mazes can be solved rapidly by digital computers, some of the types of problems approachable using fluidic mazes probably cannot. By varying the shapes and dimensions of the channels, by choosing different filling and tracing fluids, by introducing one-way paths (using flipper valves, for example), by using non-Newtonian fluids, and by using multiple entry points into the maze, this technique, we believe, will provide the possibility of generating simulations of complex systems. Using a shear-thinning non-Newtonian fluid is likely to result in a system that behaves similarly to one in which a low viscosity fluid has been injected into a high viscosity filling fluid; using a shear-thickening non-Newtonian fluid is likely to induce a behavior in the system similar to injecting a high viscosity fluid in a low viscosity filling fluid. The systems will have to be carefully designed so the shear experienced by the fluids falls within the region that induces their nonlinear behavior.

By adjustment of the properties of the channels and of the fluids used, the behavior of the fluids in the microchannels might be complex enough to model traffic flows, distribution systems, logistic chains, and similar nonlinear, complex problems. This work used high pressures to limit the influence of capillary forces at the interface when two different fluids were used, but capillary forces could be exploited to generate flows that exhibit more complex behaviors than the purely pressure-driven flow observed in these experiments. Patterned surface treatments could control the shape of the meniscus and the resulting capillary force would add or subtract from the total force applied on the moving fluids as they proceed along the microchannels. With the use of valves or magnetorheologic fluids, we can create dynamic mazes (mazes where the configuration of the channels changes with time). These techniques could also be applied to 3D mazes and mazes in which branching nodes allow multiple entrance and exit paths.

We believe that microfluidic systems may offer alternatives to electronic computing for certain types of problems. The fabrication of very complex systems of channels is straightforward, and solutions require seconds to minutes. Networks of microchannels can accommodate nonlinearities in the problems they explore, a task that is difficult for computational algorithms to fulfill. Fluidic computation can be used in conditions hostile to electronics (high temperatures or intense magnetic fields, for example), and it allows the modification of the behavior of a system without the need to change its geometry, by using fluids with different properties. There are a variety of fluidic logic elements in use,[24] and adapting those for microfluidics, and integrating them with a microfluidic system, might be more efficient than using an electronic processor for some applications.

Acknowledgment. This work was supported by grants to G.M.W. from DARPA, NIH (GM51559), and the U.S. Department of Energy (Award No. OOER45852). P.D.

(24) (a) Annino, R.; Gonnord, M. F.; Guiochon, G. *Anal. Chem.* **1979**, *51*, 379. (b) Wade, R. L.; Cram, S. P. *Anal. Chem.* **1972**, *44*, 131. (c) Standen, G.; Fortier, R. *Chem. Technol.* **1972**, *2*, 620. (d) Thorsen T.; Maerkl, S. J.; Quake, S. R. *Science* **2002**, *298*, 580.

received a fellowship from le Fond pour la Formation de Chercheur et l'Avancement de la Recherche (FCAR, Québec, PQ). We thank Scott T. Brittain for taking pictures that were used in this paper and Professor Howard A. Stone (Harvard University, DEAS) for helpful discussions on the expected behavior of fluids in microchannels.

LA030054X