

*Supporting information for*

**Warning Signals for Eruptive Events in Spreading Fires**

Jerome M. Fox and George M. Whitesides

<sup>1</sup>Department of Chemistry & Chemical Biology, Harvard University, Cambridge, MA 02138, USA.

<sup>2</sup>Wyss Institute for Biologically Inspired Engineering, Harvard University, Cambridge, MA 02138, USA.

<sup>3</sup>The Kavli Institute for Bionano Science and Technology, Harvard University, Cambridge, MA 02138, USA.

\*To whom correspondence should be addressed. E-mail: [gwhitesides@gmwgroup.harvard.edu](mailto:gwhitesides@gmwgroup.harvard.edu)

## SI Methods

**Experimental setup.** We constructed our model system by placing strips of nitrocellulose (VWR, Nitrocellulose Membrane BA83, 0.2  $\mu\text{m}$  pore size) on a suspended wire mesh and igniting them from one end (the material of this mesh did not, via conduction, influence rates of propagation for structured and unstructured flames; see SI Note 1). By imaging the propagation of flames along these strips, we examined structured and unstructured propagation behaviors (and transitions between them). The particular brand of nitrocellulose that we employed was electrostatically charged and, thus, stuck to the surface of wire mesh. To ensure adequate airflow to the bottom of burning strips, we separated the wire mesh from a glass surface by a 2.5-cm gap (Fig. S1A). To prevent stray currents of air from interfering with spread behaviors, we placed the paper/mesh setup in an enclosed fume hood with no airflow (Fig. S1B).

The meshes employed in this study consisted of two sizes (McMaster-Carr): a small mesh, which contained diamond-shaped holes with areas of 0.051  $\text{cm}^2$  and an areal density of 5 holes/ $\text{cm}^2$ ; and a large mesh, which contained diamond-shaped holes with areas of 0.13  $\text{cm}^2$  and an areal density of 2 holes/ $\text{cm}^2$ . We used the large mesh for all experiments except those indicated (e.g. Table S1). We carried out all experiments in ambient laboratory conditions maintained at 20° +/- 2° C and 12% +/- 2% relative humidity (RH) by the building environmental controls.

**Imaging.** We imaged propagating flames with a combination of high-speed (Phantom V310), thermal (FLIR), and digital (Nikon D1500) photography. For high-speed imaging, we used a capture speed of 7000 frames/second and an exposure time of 114  $\mu\text{s}$ . For thermal and digital images, we used automatic exposure and focusing. Still images of high-speed videos in which flames moved right-to-left were flipped to show flames moving left-to-right.

**Measurement of combustion rates.** To quantify rates of combustion associated with each regime, we measured the time required for a propagating flame to travel specified distances (12-20 cm) by using (i) video collected by a camera (Nikon D1500) placed directly above the wire mesh, (ii) a digital timer, and (iii) Equation S1, where  $\rho_{strip}$  is the density of the strip ( $\sim 4.9$  mg/cm<sup>2</sup>),  $v$  is the velocity of the flame (cm/s), and  $w_{strip}$  is the width of the strip (cm).

$$Rate\ of\ combustion = \rho_{strip} \cdot v \cdot w_{strip} \quad (S1)$$

To measure rates of propagation in the unstructured regime, we triggered early structured-to-unstructured transitions by placing a bump (1-cm inverted “V”) at a distance of 4 cm from the ignition end of the strip; these early transitions allowed us to image unstructured flames propagating along 20-cm distances. In Figure 2A, each data point represents an average rate of combustion calculated from at least five individual experiments.

For strips positioned at steep angles (Fig. 3B), unstructured flames moved too quickly for the accurate determination of velocities with the Nikon D1500 camera. Thus, to calculate all rates of combustion shown in Fig. 3B, we employed a high-speed camera (Phantom V310). Each data point for structured flames corresponds to the average rate of combustion calculated from at least 7 individual experiments; each data point for unstructured flames, which exhibited greater variability in velocity than structured flames, corresponds to the average rate of combustion calculated from at least 15 individual experiments.

**Probabilities of transitions.** We calculated values of  $P_{SU}$  and  $P_{US}$  for different sets of conditions (e.g. Figs. 3A and 4C) by employing the same general formula: for a particular contour and set of conditions, we counted the number of contour-initiated transitions, and then calculated  $P_{SU}$ , the probability of a structured-to-unstructured transition, or  $P_{US}$ , the probability an unstructured-to-

structured transition, by employing Equation S2, where  $n_{trans}$  is the number of successful transitions, and  $n$  is the total number of experiments ( $n \geq 25$  for this study).

$$P = \frac{n_{trans}}{n} \quad (\text{S2})$$

We calculated probabilities associated with structured-to-unstructured transitions by examining the propagation of structured flames along 30-cm strips of nitrocellulose with bumps located 15 cm from their extinction end (e.g. for strips containing bumps shaped like an inverted “V”, those bumps were placed 15 cm from the point of extinction and, thus, 13 cm from the point of ignition). We calculated probabilities associated with unstructured-to-structured transitions by examining the propagation of unstructured flames along 30-cm strips of nitrocellulose with bumps located 4 cm from the ignition end of the strip (to initiate the unstructured flame) and 8 cm from the extinction end of the strip (to trigger the unstructured-to-structured transition).

**Calculation of mean apparent brightness.** We examined dynamics of our system by monitoring the temporal evolution of the mean apparent brightness ( $B_{ap}$ ) for successive high-speed images. To evaluate values of  $B_{ap}$ , we wrote a MATLAB program that carries out the following steps: (i) it calculates the mean values of red, green, and blue pixel intensities ( $I_{red}$ ,  $I_{green}$ , and  $I_{blue}$ , respectively) within the 512x512 uint8 matrix describing those intensities for each image; (ii) it calculates  $B_{ap}$  by adding those values together ( $B_{ap} = I_{red} + I_{green} + I_{blue}$ ); (iii) it carries out (i) and (ii) for successive frames; (iv) it calculates the matrix of  $(t, B_{ap})$  pairs and plots them as shown in Fig. 4B and Figs. S9-S10.

**Statistical indicators of slowing down.** To evaluate commonly proposed statistical indicators of critical slowing down, we calculated the variance, lag-1 autocorrelation, and skewness of  $B_{ap}$  for structured flames propagating along strips positioned at different angles relative to a level

surface ( $\theta_{strip}$ ). In order to examine patterns in behavior that emerge as structured-to-unstructured transitions become more likely, we chose angles within the bistable region (Fig. 2C) located at different proximities to  $\theta_{strip} = 115^\circ$  (the structured-to-unstructured bifurcation point). To carry out the required calculations, we wrote a MATLAB program that implemented the operations described below for 7000 frames (1 second) within ten separate videos at each angle (i.e. for each data point in Fig. 4A).

Variance: The program calculated the variance of  $B_{ap}$  (a measure of the spread of  $B_{ap}$  about the sample mean) by implementing Equation 1, where  $n$  is the total number of frames (7000),  $B_i$  is the mean apparent brightness of frame  $i$ , and  $\bar{B}$  is the mean apparent brightness of all 7000 frames.

$$Var(B_{ap}) = \frac{1}{n-1} \sum_{i=1}^n (B_i - \bar{B})^2 \quad (1)$$

Lag-1: The program calculated the lag-1 autocorrelation (Lag-1 AC; a measure of the self-similarity of  $B_{ap}$  over time) by implementing Equation 2(I), where  $n$  is the size of the window ( $n=7000$ ),  $B_i$  is the brightness of frame  $i$ , and  $\bar{B}$  is the mean apparent brightness of all 7000 frames.

$$Lag_1 AC(B_{ap}) = \frac{\frac{1}{n} \sum_{i=1}^{n-1} (B_i - \bar{B})(B_{i+1} - \bar{B})}{\frac{1}{n-1} \sum_{i=1}^n (B_i - \bar{B})^2} \quad (2)$$

Skewness: The program calculated the skewness of  $B_{ap}$  (a measure of the asymmetry in the distribution of  $B_{ap}$  about the mean) by implementing Equation 3, where  $B_i$  is the mean apparent brightness of frame  $i$ , and  $\bar{B}$  is the mean apparent brightness of all 7000 frames.

$$Skewness(B_{ap}) = \frac{\frac{1}{n} \sum_{i=1}^n (B_i - \bar{B})^3}{\left( \sqrt{\frac{1}{n} \sum_{i=1}^n (B_i - \bar{B})^2} \right)^3} \quad (3)$$

**Behavioral indicators of slowing down.** To estimate values of  $t_{recovery}$ , the time between the end of an encounter with a bump and the return of structured behavior, we carried out the following steps: (i) we used high-speed videos, run at 1/70 actual time, to identify the first frame where no part of the bump remained (complete combustion), and (ii), we ran a MATLAB program calculated the time between that frame and the first frame where the local average of  $B_{ap}$  (calculated over 1000 frames) reached a value of within 5% of the average value of  $B_{ap}$  in the structured regime (calculated over the first 7000 frames, well before the bump).

**Image analysis and particle tracking.** To estimate the velocity of gases within nitrocellulose-fed flames, we developed a MATLAB program to determine the trajectory and speed of incandescent nitrocellulose particles within each flame. For each of 500 sequential images in a high-speed video of a propagating flame, this program (i) carried out a real space bandpass filter with noise and length settings optimized for the detection of incandescent particles of nitrocellulose, (ii) determined the centroid of each particle, (iii) tracked particles between adjacent flames, and (iv) determined the velocity of particles that could be tracked over ten frames (Fig. S7). The velocities used to calculate values of  $F_{thrust}$  and  $Fr$ , described below, represent the average of the ten fastest velocities for any given image set; these correspond the velocities of particles traveling close to the surface of the nitrocellulose strips.

**Estimation of Froude numbers ( $Fr$ ).** For flames, the Froude number ( $Fr$ ) represents a metric for the relative contribution of inertial forces to buoyant forces to flame structure (Eq. S3)(2); in Equation S3,  $U$  is the velocity of gases within the flame,  $g$  is the acceleration due to gravity, and  $W$  is the width of the flame (or, in this study, the width of the strip).

$$Fr = \frac{U^2}{g \cdot W_{strip}} \quad (S3)$$

We estimated values of  $Fr$  for flames of different widths by employing image analysis (described above) to estimate  $U$ .

## **SI Notes**

### **SI Note 1. An examination of transport phenomena influencing rates of propagation.**

For the structured regime, we assessed the contributions of (i) conductive heat flux into the support surface and (ii) oxygen flux out of the support surface by measuring rates of combustion along surfaces that differed in their thermal diffusivities and/or ability to permit oxygen transport to the bottom of the strip (Fig. S2). Rates of combustion proved insensitive to the composition of the support surface, but increased as restrictions to oxygen transport were lifted (no mesh to small mesh to large mesh). This last result, in light of the linear relationship between the rate of combustion and the width of the strips, suggests that restrictions on oxygen transport existed, but scaled with the width of the strips (and thus did not become more or less important as width was increased).



## SI Note 2. Measurement of the angle of propagating flames.

To assess how movements of the nitrocellulose strips could cause forward-bursting blasts (Fig. S3A), we tilted strips at various angles relative to a level surface ( $\theta_{strip}$ ), and examined the orientation of flames propagating along them (Fig. S3B).

We measured  $\theta_{flame}$ , the angle between the flame and the surface of a strip, by (i) collecting high-speed images of structured flames propagating along strips positioned at various angles relative to a level surface (i.e. different values of  $\theta_{strip}$ ), and by (ii) using an angle measurement function of ImageJ to determine the angle between the front of the flame and the surface of the strip. The electrostatically charged strips stuck to the surface of the wire mesh for all values of  $\theta_{strip}$ .

A plot of  $\theta_{flame}$  vs.  $\theta_{strip}$  shows that the orientation of a flame is dominated not by gravity, but by the inertia of gases leaving the strip and, thus, by the orientation of the strip (Fig. S3C). This result suggests that movements of the strip generate forward blasts of hot gases by tilting flames in the forward direction.

### SI Note 3. Estimation of buoyancy and thrust.

For a 2.54-cm wide strip, we estimated the buoyant force that a structured flame burning on the underside of a strip exerts upward on that strip by carrying out the following steps: (i) we estimated  $T_{flame}$ , the average temperature of gases within the flame, by using thermal images; (ii) we estimated  $\rho_{flame}$ , the density of gases within the flame, by employing the ideal gas law (Eq. S4, where  $T_{air}$  is the temperature of ambient air in the room and  $\rho_{air}$  is the density of the bulk air);

$$\rho_{flame} = \frac{T_{air}}{T_{flame}} \rho_{air} \quad (S4)$$

(iii) we estimated  $V_{flame}$ , the volume of the flame beneath the strip, by using front and side images of the flame; and (iv) we estimated the buoyant force by employing Equation S5, where  $g$  is the acceleration due to gravity.

$$F_{buoyancy} = (\rho_{air} - \rho_{flame}) \cdot g \cdot V_{flame} \quad (S5)$$

Values of  $V_{flame}$  and  $\rho_{fire}$  (Table S2) represent averages of measurements collected from at least three separate experiments. Fig. S5 shows images representative of those used to calculate (i)  $V_{flame}$  and (ii)  $T_{flame}$  (and, thus,  $\rho_{flame}$ ).

For a 2.54-cm wide strip, we estimated the thrust that a structured flame burning on the underside of a strip exerts upward on that strip by multiplying the velocity of gases emanating from the surface of the strip by the mass flow rate of those gases (Eq. S6, where  $U$  is the velocity of incandescent particles of nitrocellulose within a structured flame burning upside down,  $\rho_{strip}$  is the density of the strip in  $\text{g/cm}^2$ ,  $v$  is the velocity of the propagating flame in  $\text{cm/s}$ , and  $w_{strip}$  is the width of the strip in  $\text{cm}$ ).

$$F_{thrust} = U \cdot \dot{m} = U \cdot (\rho_{strip} \cdot v \cdot w_{strip}) \quad (S6)$$

At  $\theta_{strip} = 180^\circ$ , the structured flame was not stable and, thus, could only be imaged propagating over short distances; to facilitate measurement of its velocity, we used high-speed photography.

Final values of  $U$ ,  $\rho_{strip}$ , and  $v$  (Table S2) represent averages of measurements collected from at least three separate experiments. Fig. S5 shows images representative of those used to measure  $U$  and  $v$ .

Using the aforementioned approaches, we calculated values of  $F_{buoyancy}$  and  $F_{thrust}$  of  $\sim 50$  and  $\sim 140 \mu\text{N}$ , respectively. If we neglect drag on the paper strip and assume constant acceleration, we can calculate the time required for these forces to move a 1-cm stretch of strip over a distance of 1 cm by employing Equation S7, where  $\Delta x$  is the displacement (1 cm), and  $m$  is the mass of a 1-cm stretch of 2.54-cm wide strip.

$$t = \sqrt{\frac{2 \cdot \Delta x}{(F_{buoyancy} + F_{thrust})/m}} \quad (\text{S7})$$

Using the values in Table S2, we calculate  $t$  to be  $\sim 36$  ms, a time similar to that over which we observe 1-cm movements of strip in videos (20-50 ms).

#### **SI Note 4. Forces responsible for causing unstructured flames to shift their positions.**

Back-and-forth movements of nitrocellulose strips suggest that unstructured flames shift their positions dynamically, constantly pushing the strips in different directions. High-speed videos of structured flames encountering the bump (1-cm inverted “V”), which exerts tension on the strip, reveal the forces responsible for such shifts (Movies S3 and S5). A 1-cm inverted “V” consists of three folds: one where the strip meets the front of the bump (fold 1, leftmost fold in Fig. 1B), one at the center of the bump (fold 2, central fold Fig. 1B), and one where the strip meets the back of the bump (fold 3, rightmost fold Fig. 1B); folds 1 and 3 exert tension on the strip (as they are folded in our procedure), tugging it upward. Prior to ignition of the strip, this tension is counteracted by the weight of the strip; as the flame burns away the mass holding the strip down, however, the tension exerted by fold 1 eventually pulls the strip upward (Fig. 4D, 54 ms). As the strip moves away from the mesh, drag forces exerted by the air on the flame push it to the underside of the strip (Fig. 4D, 99 ms). As the flame moves closer to the bump, buoyancy allows the flame to curl around to the top of the strip again (Fig. 4D, 658 ms). Thus, in this system, flames move from one side of a strip to the other via (i) drag forces (i.e. wind, relative to the reference flame of the moving strip) and (ii) buoyant forces (in the case of bottom-to-top movements), thereby pushing strips in different directions.

### **SI Note 5. Mechanical stiffness and the unstructured regime.**

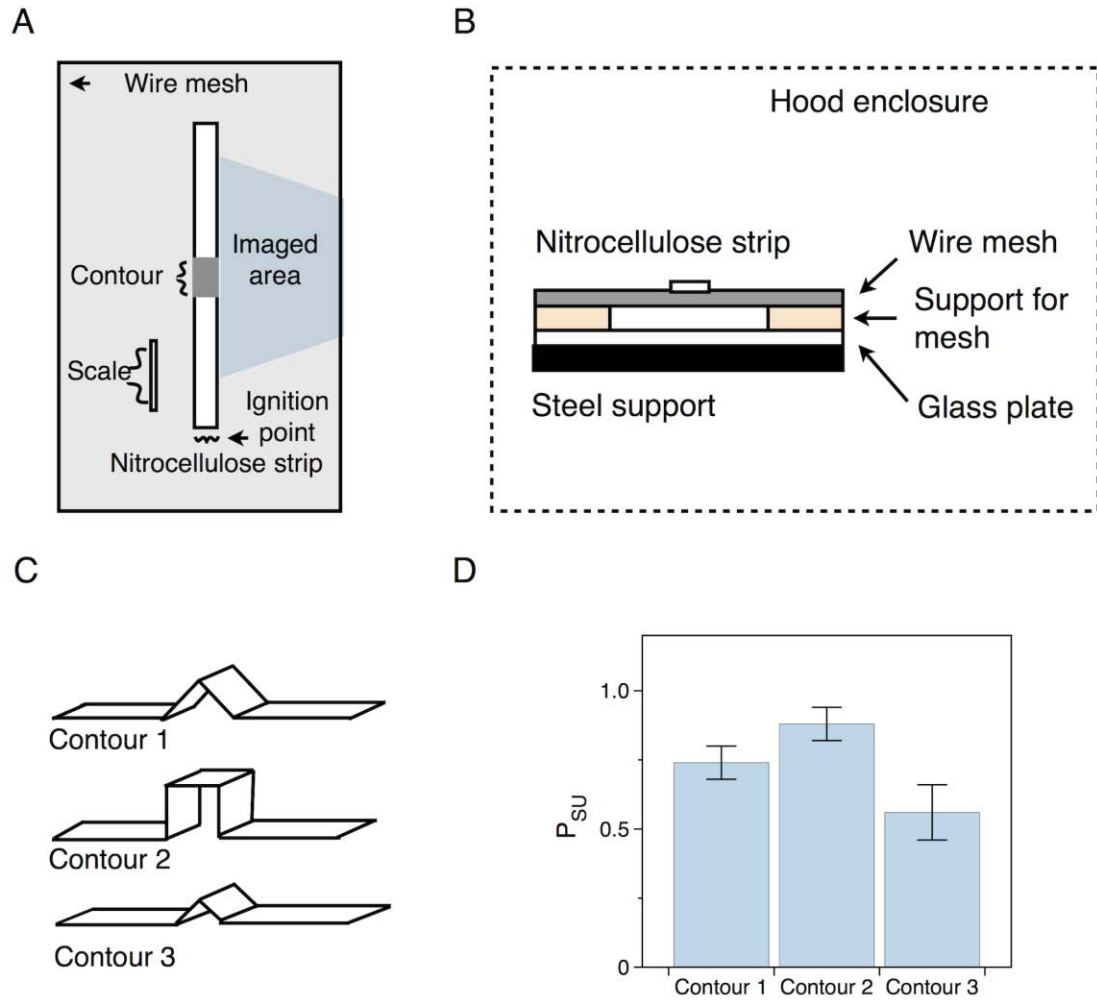
In the main text, we concluded that in the unstructured regime, propagating flames drive movements of the nitrocellulose strip and simultaneously shift their position in response to wind generated by those movements. To further examine this feedback loop, we tested the ability of structured flames to undergo structured-to-unstructured transitions while propagating along double-layer stacks (i.e. stacks of two strips, each containing a 1-cm inverted “V”). We hypothesized that these stacks, which are twice as thick—and, thus, twice as stiff—as single-layer strips, might be less responsive (in terms of movement) to the thrust and buoyant forces generated by unstructured flames; accordingly, this stiffness might obstruct the feedback loop that stabilizes the unstructured regime. To test this hypothesis, we first measured  $P_{SU}$  for the double-layer stack containing 1-cm bumps. Over 25 experiments, we observed no successful structured-to-unstructured transitions ( $P_{SU} = 0$ ); we then examined high-speed videos for encounters under these conditions (Movie S6). These videos show that double-layer strips do, in fact, exhibit far less movement throughout the encounter than single-layer strips. These results are consistent with our hypothesis and suggest that the flexibility and energy density (lightness of the material supporting a flame of a given size) of nitrocellulose strips are key components of the feedback loop that stabilizes the unstructured regime.

## **SI Note 6. Examination of oscillatory behavior.**

We examined the source of oscillations in  $B_{ap}$  by comparing plots in  $B_{ap}$  for structured and unstructured flames with corresponding high-speed videos and images. Figures S9A-9D show the different patterns of fluctuations associated with structured and unstructured flames. Figure S9A shows the stochastic fluctuations of a structured flame. Figure S9B shows oscillations resulting from oscillating asymmetry in the flame front. These oscillations are elaborated in Figures S9E-S9G. We define “oscillating asymmetry” to mean the oscillating angles between the wide dimension of the flame and the long dimension of the strip; by affecting the area of the flame exposed to the camera, these angles affect values of  $B_{ap}$ . Figure S9C shows oscillations resulting from the formation of convection cells that cause undulations in the height of the flame. Movie S1 shows these oscillations at 1/70 their actual speed (they also appear in the beginning of Movie S3). Figure S9D shows aperiodic oscillations associated with the random movements of unstructured flames. Figure S10 shows different patterns of fluctuations in  $B_{ap}$  for structured flames; plots correspond to nine experiments conducted under the same set of conditions. When we refer to “irregular oscillatory behavior” in reference to structured flames in the main text, we are referring to the transient, irregular, or unpredictable appearance of oscillations caused by oscillating asymmetry in the flame front and/or by the formation of convection cells. Such oscillations appear in some, but not all, of the plots in Figure S10.

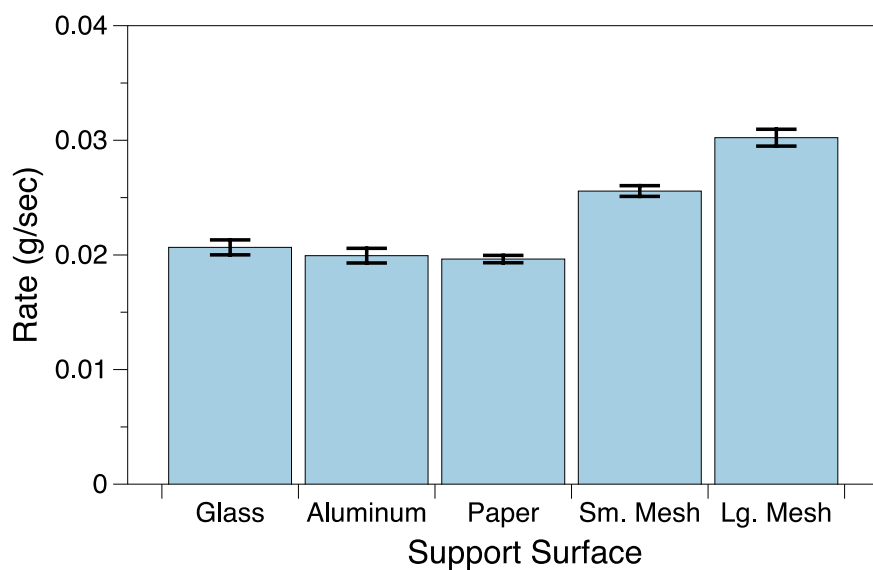
## SI References

1. Box GEP, Jenkins GM, Reinsel GC (1994) *Time Series Analysis: Forecasting and Control* (Prentice Hall, Englewood Cliffs, NJ). 3rd Ed.
2. Drysdale D (1999) *An Introduction to Fire Dynamics* (Wiley, New York).

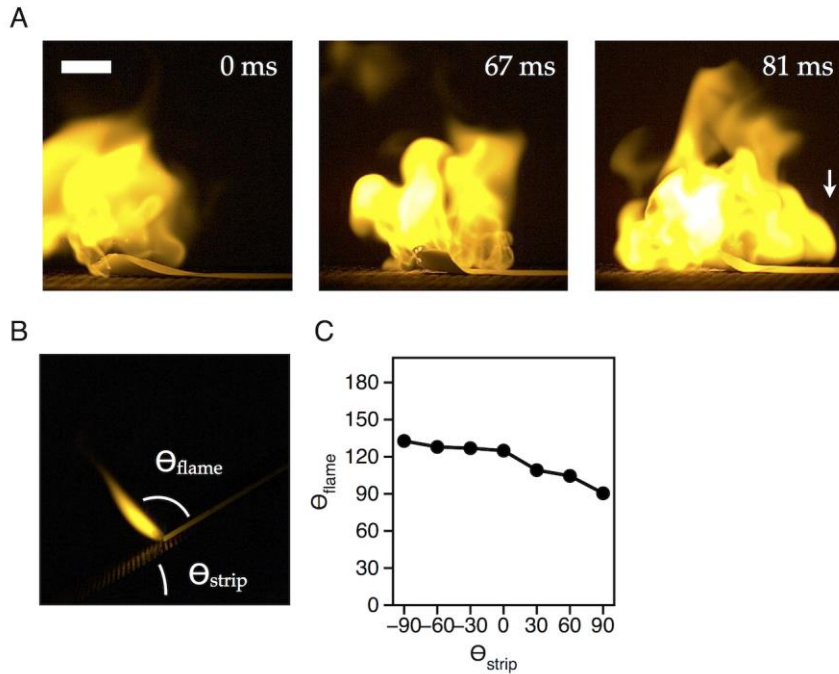


**Fig. S1.** Experimental setup. (A) We placed strips of nitrocellulose (with a length of 30 cm, a thickness of 140  $\mu\text{m}$ , and widths ranging from 0.5 to 5 cm) onto a suspended wire mesh, and ignited them from one end. A 10-cm tall piece of cellulose paper, present in every image (scale), allowed us to determine pixel/length calibrations; from these calibrations, we added scale bars to each image. (B) Plastic petri dishes (pink) held the corners of the wire mesh 2.5 cm above a glass support surface (white), and an airflow-free fume hood (dashed lines) minimized interference by stray convective currents. (C) Several different bumps: a 1-cm inverted “V” (contour 1), a 1-cm square bump (contour 2), a 0.5-cm inverted “V” (contour 3). (D) The probability of a structured-to-unstructured transition for the three contours shown in (C). Error bars indicate the standard error of the probability ( $n \geq 25$ ).

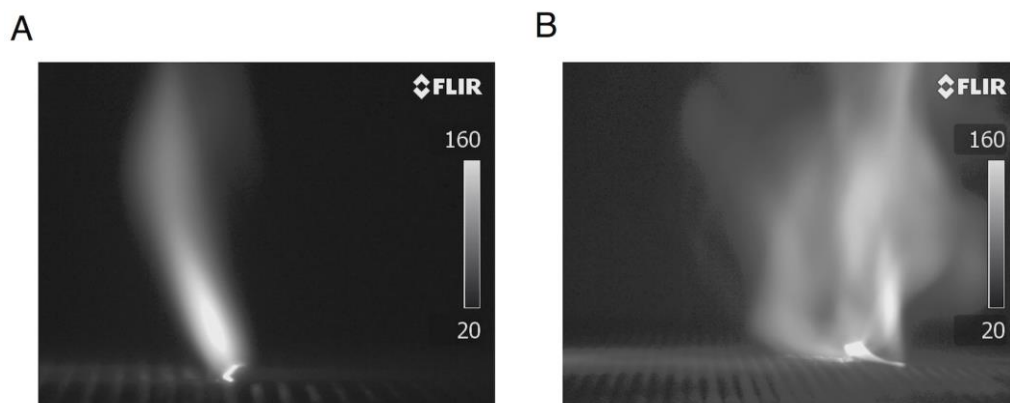




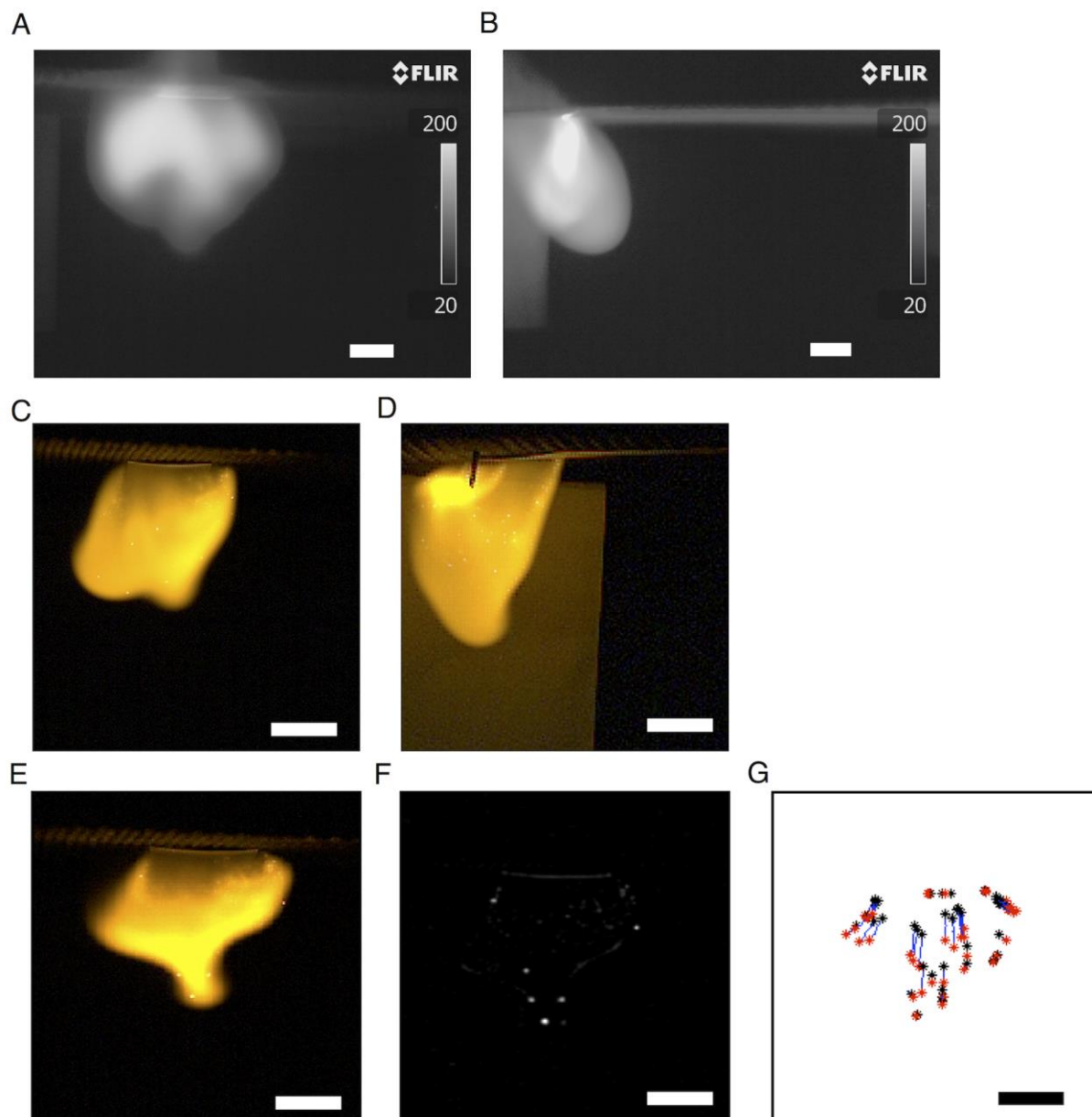
**Fig. S2.** Rates of combustion for structured flames propagating along strips supported by different materials. Flames traveled along 2.54-cm strips. The small mesh contains diamond-shaped holes with areas of  $0.051 \text{ cm}^2$  and a density of  $\sim 5 \text{ holes/cm}^2$ . The large mesh contains diamond-shaped holes with areas of  $0.13 \text{ cm}^2$  and a density of  $\sim 2 \text{ holes/cm}^2$ . The holes account for  $\sim 25\%$  of each mesh (by area). Error bars represent standard error ( $n \geq 5$ ).



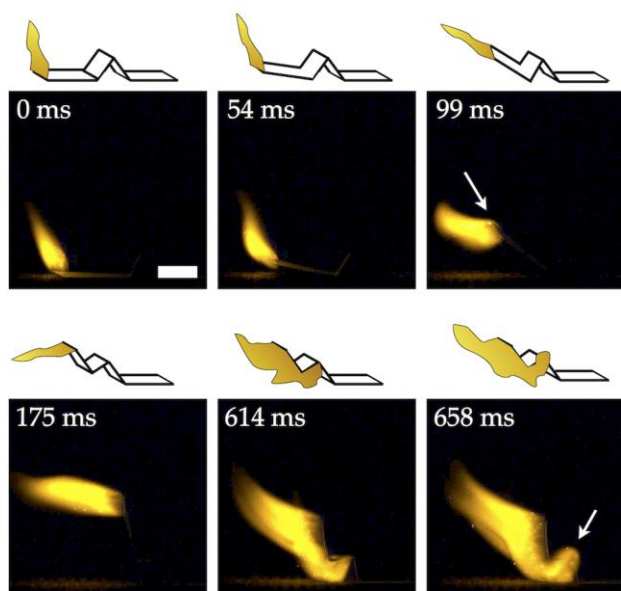
**Fig. S3.** (A) Convective blasts generated by movements of the strip. Still frames from a high-speed video of an unstructured flame show how movements of the strip drive blasts of hot gases that extend forward over the surface of the strip (white arrow). Scale bar = 2 cm. (B) A high-speed image showing a structured flame propagating at  $30^\circ$  relative to a level surface ( $\theta_{strip} = 30^\circ$ ). (C) Relationship  $\theta_{strip}$  and the angle between the flame and the surface of the strip ( $\theta_{flame}$ ).



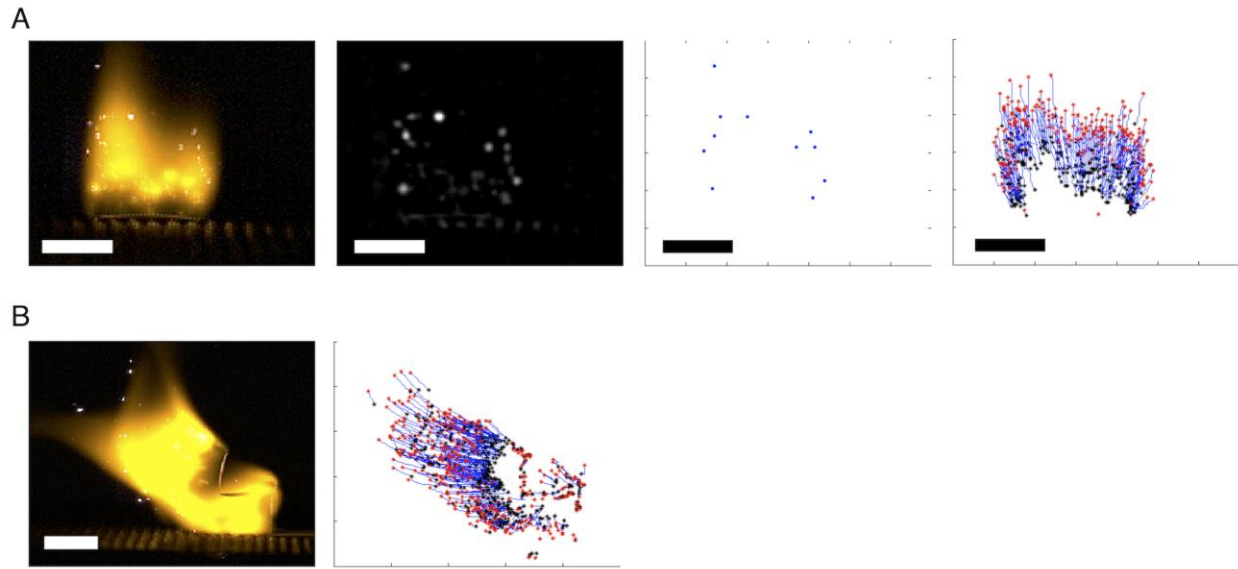
**Fig. S4.** Thermal images of structured and unstructured flames. (A) An infrared image of a structured flame propagating along a 2.54-cm wide strip. Hot gases extend beyond a sharply delineated region of ignited nitrocellulose (large grid). (B) An infrared image of an unstructured flame traveling along a 2.54-cm wide strip (large grid). Hot gases extend far beyond a region of ignited nitrocellulose that is much larger than the corresponding region of the structured burning regime. The temperature scale was fixed at a maximum of 160°C to optimize the visibility of hot gases within the flames.



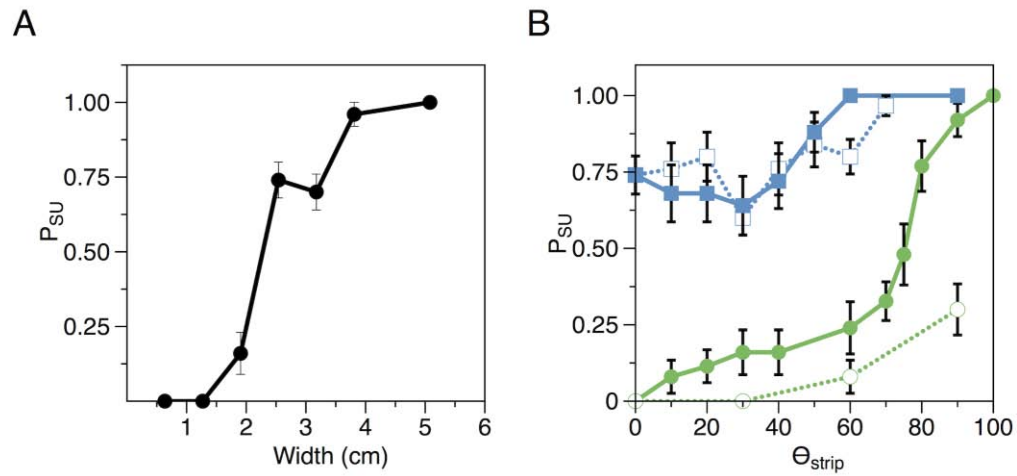
**Fig. S5.** Images of a structured flame burning upside down. For (A-G),  $\theta_{\text{strip}} = 180^\circ$  and  $W_{\text{strip}} = 2.54\text{-cm}$ . (A) Thermal image of the front of the flame (propagating away from the camera). (B) Thermal image of the side of the flame (propagating from left to right). (C) High-speed image of the front of the flame (propagating away from the camera). (D) High-speed image of the side of the flame (propagating left to right). (E) High-speed image of the front of the flame. (F) Filtered image of the front of the flame showing incandescent particles of nitrocellulose. (G) Particle trajectories. (A-G) Scale bars = 1 cm.



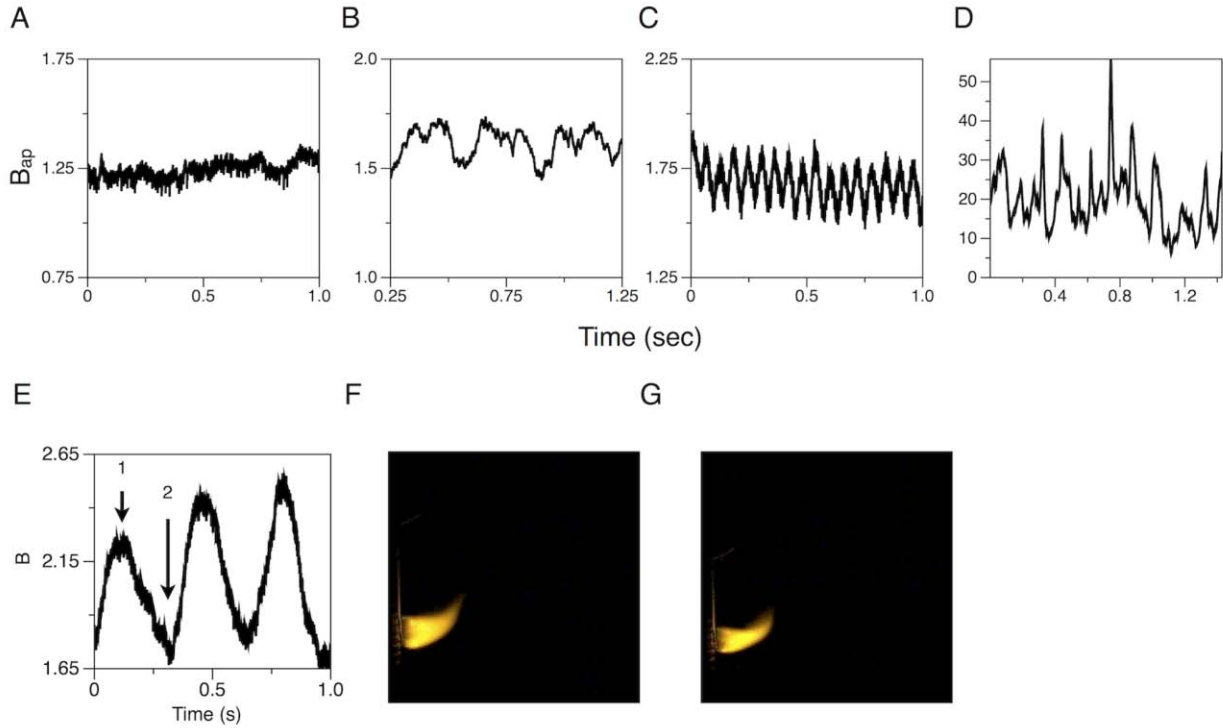
**Fig. S6.** High-speed images of a structured flame encountering a 1-cm inverted “V”. As the flame approaches the bump, tension in the bump raises the paper up (54 ms), exposing the flame to wind, which pushes it to the underside of the strip (99 ms, white arrow). As the flame spreads further into the bump (614 ms), buoyant forces push part of the flame back to the top of the strip (658 ms, white arrow).



**Fig. S7.** Details of image analysis. (A) Steps for calculating the velocity of gases within nitrocellulose-fed flames: we captured high-speed images of structured flames (left), subjected those images to a real-space bandpass filter optimized to isolate nitrocellulose particles (left middle), identified the centroid of each particle (right middle), and plotted the trajectories of those particles over 500 successive frames (right). (B) A structured-to-unstructured transition (scale bar = 1 cm): high-speed image (left), trajectories of particles (right).

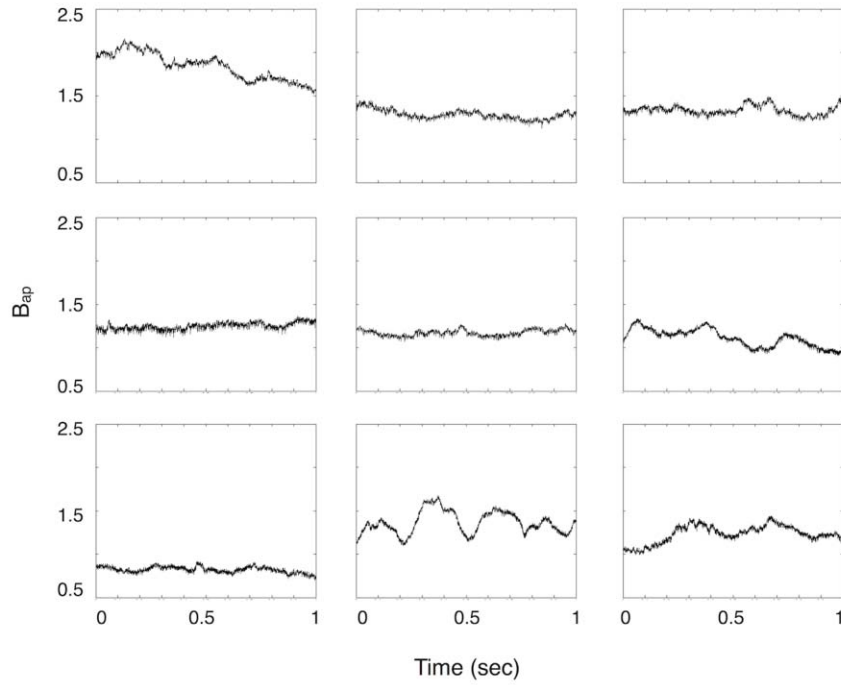


**Fig. S8.** The influence of width of the strips and slope of the mesh supporting them on  $P_{SU}$ . (A) A plot showing values of  $P_{SU}$  for flames propagating along strips of different widths. (B) A plot showing values of  $P_{SU}$  for flames propagating up (solid lines) and down (dotted lines) strips tilted at various angles relative to a level surface ( $\theta_{strip}$ ). Colors correspond to different widths of strip: 2.54 cm (blue) and 1.27 cm (green). Error bars represent standard error ( $n \geq 25$ ).



**Fig. S9.** Examples of fluctuations in  $B_{ap}$ . (A-D) Mean values of  $B_{ap}$  for sequential high-speed images of (A) a structured flame, (B) a structured flame exhibiting periodic oscillations caused by oscillating asymmetry in the flame front (1.27-cm strip, large mesh, 90°C), (C) a structured flame exhibiting periodic oscillations caused by the formation of convection cells (2.54-cm strip, small mesh), and (D) an unstructured flame exhibiting aperiodic oscillations caused by cyclic convective blasts of hot gases (2.54-cm strip, small mesh). (E) The mean apparent brightness ( $B_{ap}$ ) of a structured flame propagating along a 1.27-m strip positioned at 90° relative to a level surface ( $\theta_{strip} = 90^\circ$ ). (F) A high-speed image of the flame at position 1 in (E). (G) A high-speed image of the flame at position 2 in (E). (F-G) As the angle between the wide dimension of the flame and the long dimension of the strip changes, the area of the flame exposed to the camera, which is reflected in the magnitude of  $B_{ap}$ , responds accordingly.





**Fig. S10.** Variability in  $B_{ap}$  for a single set of conditions. Plots show the evolution of mean apparent brightness ( $B_{ap}$ ) of a structured flame propagating up a 1.27-cm strip positioned at  $40^\circ$  relative to a level surface ( $\theta_{\text{strip}} = 40^\circ$ ). Variability between the plots arises from the irregular nature of oscillatory fluctuations.

**Table S1. Probabilities of transitions under different conditions.**

<b>Condition</b>	<b>Transition<sup>*</sup></b>	<b>Width of strip (cm)</b>	<b>Probability</b>
Large Mesh <sup>**</sup> (20°C)	SU	1.27	0
	SU	2.54	0.74(0.06)
	SU	3.81	0.96(0.04)
	US	2.54	0.04(0.04)
Small Mesh <sup>**</sup> (20°C)	SU	2.54	0.68(0.07)
	SU	3.81	0.72(0.09)
	US	2.54	0.22(0.07)
Large Mesh (90°C)	SU	1.27	0.36(0.1)
	SU	2.54	1.0
Glass (20°C)	SU	1.27	0
	SU	2.54	0.04 (0.04)

<sup>\*</sup>SU: structured-to-unstructured, US: unstructured-to-structured.

<sup>\*\*</sup>Large and small meshes are described in detail in Methods.

**Table S2. Parameters used estimate thrust and buoyancy.**

<b>Parameter</b>	<b>Estimated Value</b>
$U$	271(61) cm/sec
$\rho_{strip}$	4.9 (0.1) g/cm <sup>2</sup>
$v$	4.0(0.5) cm/s
$T_{air}$	20 °C
$T_{flame}$	200 °C
$\rho_{air}$	1.21
$\rho_{flame}$	0.75
$g$	9.82 cm/s <sup>2</sup>
$V_{flame}$	12.05 cm <sup>3</sup>
$F_{thrust}$	137 μN
$F_{buoyancy}$	54 μN

## **SI Movie Legends.**

In all videos, the thin strip of paper on the left side of the screen functions as a 10-cm scale bar.

**Movie S1. A structured flame (1/70 actual speed).** In this video, a structured flame propagates along a 2.54-cm wide strip of nitrocellulose (large mesh).

**Movie S2. An unstructured flame (1/70 actual speed).** In this video, an unstructured flame propagates along a 2.54-cm wide strip of nitrocellulose. A 1-cm triangular bump, positioned 4 cm from the point of ignition, generated this flame via a structured-to-unstructured transition.

**Movie S3. A structured-to-unstructured transition (1/10 actual speed).** A 1-cm triangular bump, positioned 13 and 15 cm from the ignition and extinction ends of a 2.54-cm wide strip (large mesh), respectively, triggered this transition. Available online at:

**Movie S4. An unstructured-to-structured transition (1/10 actual speed).** A 1-cm triangular bump, positioned 4 cm from the ignition end of a 2.54-cm wide strip (small mesh), triggered the initial unstructured flame. A second bump, positioned 14 cm away from the first (8 cm from the extinction end of the strip) triggered the unstructured-to-structured transition.

**Movie S5. An structure-to-unstructured transition followed by an unstructured-to-structured transition (1/10 actual speed).** A 1-cm triangular bump, positioned 13 cm away from the ignition end of a 2.54-cm wide strip (small mesh) triggered the first transition (structured-to-unstructured). A second bump, positioned 5 cm away from the first (and 8 cm away from the end of the strip), triggered the second transition (unstructured-to-structured).

**Movie S6. A failed structured-to-unstructured transition on a double-layer strip (1/10 actual speed).** A flame propagating along a stack of two 2.54-cm wide strips (large mesh) containing a 1-cm triangular bump.