Magnetic Levitation in Chemistry, Materials Science, and Biochemistry

Supporting Information

Shencheng Ge,[1] Alex Nemiroski, [1] Katherine A. Mirica, [1] Charles R. Mace, [1] Jonathan W. Hennek,[1] Ashok A. Kumar, [1] and George M. Whitesides[1,2,3*]

[1] Department of Chemistry & Chemical Biology, Harvard University, 12 Oxford Street, Cambridge, MA 02138, USA

[2] Wyss Institute for Biologically Inspired Engineering, Harvard University, 60 Oxford Street, Cambridge, MA 02138, USA

[3] Kavli Institute for Bionano Science & Technology, Harvard University, 29 Oxford Street Cambridge, MA 02138, USA

* Corresponding author: George M. Whitesides (gwhitesides@gmwgroup.harvard.edu)

Table of Contents

S1. Magnetism Relevant to MagLev

S1.1 Diamagnetism

Diamagnetism is a characteristic of all matter, but is only apparent in materials with no unpaired electrons. If, however, a molecule or material contains unpaired electrons, then its interactions with an applied magnetic field may dominate diamagnetism. To understand the origin of diamagnetism, consider a simplified classical picture of an electron orbiting the nucleus of an atom. As a charge in motion, this electron generates a magnetic field. When an external magnetic field is applied, the electron alters its motion to oppose the change of field (Lenz's law).[1] The consequence of this effect is induced magnetization in a substance that opposes the applied field; this molecular-level response to an applied magnetic field is called diamagnetism. The effect of diamagnetism is, thus, universal for all matter. Most organic materials are diamagnetic. Examples of organic diamagnetic materials include water, most organic liquids, typical biological polymers such as proteins (those that do not contain transition metals), DNA, and carbohydrates, and most synthetic or semi-synthetic polymers. A few representative exceptions include stable free radicals (e.g., trityl and nitroxyls), O_2, many organometallic compounds and chelates of transition metals; these may be paramagnetic (see Section S1.2).

A measure of the type and magnitude of magnetization of a material in response to an applied magnetic field is the magnetic susceptibility, χ_v (volume susceptibility, unitless), as defined in eq S1, where $\vec{\mathrm{M}}$ (A m^{-1}) is the magnetization of the material, $\vec{\mathrm{H}}$ (A m^{-1}) is the applied external magnetic field. Eq S2 describes the closely related magnetic field $\vec{\mathrm{B}}$ (T) present in a material, where μ_0 is the magnetic permeability in vacuum.

$$\chi_v = \vec{\mathrm{M}}/\vec{\mathrm{H}} \qquad \text{(S1)}$$

$$\vec{\mathrm{B}} = \mu_0(\vec{\mathrm{H}} + \vec{\mathrm{M}}) = \mu_0(1 + \chi_v)\vec{\mathrm{H}} \qquad (S2)$$

The magnetic susceptibility of typical diamagnetic materials is around -10^{-5} (in SI unit, the negative sign indicates that the induced magnetization opposes the applied field), and is essentially indistinguishable for many materials, including some of the metals and the majority of the organic materials (Figure 2). Bismuth and pyrolytic graphite are notable exceptions, and are up to two orders of magnitude more diamagnetic than common diamagnetic materials. Because the induced magnetization of diamagnetic materials is negative and small, they are slightly repelled from regions of magnetic field. This repulsion can be sufficient to suspend the most diamagnetic materials (e.g., pyrolytic graphite) against gravity in air (and even in vacuum) using permanent magnets, while it is not noticeable when common diamagnetic materials interact with the modest field of the permanent magnets in air (although they can be important with the much higher fields of superconducting magnets). The magnetic susceptibility of most diamagnetic materials is independent of temperature over commonly encountered ranges.[2]

S1.2 Paramagnetism

In one view of paramagnetism, it originates from the unpaired electrons present in the material that produces permanent magnetic moments. In the absence of an applied magnetic field, the spins of the unpaired electrons are randomly oriented in space and time due to thermal energy: background magnetic fields—a field gradient—(e.g., from the earth) introduce interactions far weaker that those from thermal motions. The presence of an external magnetic field thus tends to align (weakly) the magnetic moments in the direction of the applied field. The response of paramagnets to an applied field at room temperature is typically three orders of magnitude greater than that of diamagnets. They are attracted to, rather than repelled by, the applied field. Typical magnetic susceptibilities of paramagnets are ~ 10^{-3}–10^{-5} (in SI unit). Paramagnetic species relevant to the type of MagLev we discuss in this review include simple paramagnetic salts, such as $MnCl_2$, $GdCl_3$, $HoCl_3$, $DyCl_3$, $CuCl_2$, and $FeCl_2$, and the chelates of some of these ions (e.g., Gd^{3+}). See Section 3.3 for a more detailed discussions.

S1.3 Ferromagnetism

Ferromagnetism is the property of a material that exhibits spontaneous (and permanent) magnetic moments; that is, the material has a high magnetic moment even in the absence of an external field.[1] Ferromagnetism only occurs in materials that contain strongly interacting unpaired spins. These spins in the material interact in such a way that they align with each other in the same aligned direction in localized regions—termed magnetic domains. In the apparently "unmagnetized" ferromagnetic materials, the magnetic moments of the magnetic domains are disordered and thus, effectively cancel. An external magnetic field can align the magnetic domains in the materials, and the collective alignment of spins in magnetic domains produces a net magnetic moment. Strong magnetic moments remains in ferromagnetic materials when the applied magnetic field is removed. Common ferromagnetic materials are iron, iron oxides (e.g., magnetite), cobalt, nickel, their alloys (e.g., Alnico), and, importantly, alloys containing rare-earth metals (e.g., NdFeB and SmCo). Permanent NdFeB magnets that enable the type of MagLev we describe in this review are ferromagnetic.[3,4]

S1.4 Superparamagnetism

Superparamagnetic materials behave qualitatively similarly to paramagnetic materials in an applied magnetic field, but exhibit a much stronger response (in terms of magnetic susceptibility). Superparamagnetism exists in small ferro- or ferri-magnetic nanoparticles (especially iron oxides in the range of 3-50 nm), and they are effectively single magnetic domains. The magnetization of individual nanoparticles can flip randomly in directions due to thermal motions, and thus, they do not exhibit a net magnetization; an applied magnetic field, however, can align the magnetic moments of individual nanoparticles, and thus, produces a strong magnetic response in these materials (often much larger than paramagnetic materials). Unlike ferro- or ferrimagnetic materials, the magnetic moments of these nanoparticles are not retained upon removal of the magnetic field. This type of magnetism is not commonly used in the MagLev techniques we describe here (See Section 3.3.6 for more discussions); it is, however, perhaps the most recognized and used form of magnetism in biochemistry and biology, and is employed to separate biological entities (e.g., proteins, organelles, viruses, bacteria, and mammalian cells) using affinity-ligand-coated superparamagnetic particles, and is the basis of ferrofluids.[5–7] Table 1 compares MagLev and magnetic separations using superparamagnetic particles.

S2. Qualitative Characteristics of MagLev

S2.1 Basic Principles of MagLev

Stable levitation of a suspended diamagnetic object in a paramagnetic medium in an applied magnetic field indicates a minimum in the total energy of the system, including both the gravitational energy and the magnetic energy (Figure 1F). In the absence of an applied magnetic field, a suspended object in a medium will either sink or float to minimize the total gravitational energy, including the gravitational energy of the object and of the medium that is displaced by the object. For example, an object having a density higher than the medium ($\rho_s > \rho_m$) will always sink in a gravitational field to minimize its height, and thus, the gravitational energy of the system. The magnetic energy of the system, including the diamagnetic object and the displaced paramagnetic medium, however, has a different profile in space (See Figure 1F for the profile of U_m for the "standard" MagLev system). This magnetic energy is a function of three parameters: the volume of the object, the difference in magnetic susceptibility of the object and the suspending medium, and the strength of the magnetic field at the position the object situates in space. The magnetic field will always tend to minimize the magnetic energy by pushing the suspended object to regions in which the field strength is weaker (that is, toward the center of the "magnetic bottle"). Stable levitation of the suspended object will occur only if the *sum* of the magnetic energy and the gravitational energy of the system reaches a minimum. For stable levitation, any deviation of the object from the equilibrium position will always incur an energy cost; the object is, therefore, energetically "trapped" at this position, or levitated stably in the suspending medium. In limiting cases where the gravitational energy dominates (e.g., the sample is significantly more dense or less dense than the medium, $\rho_s \gg \rho_m$ or $\rho_s \ll \rho_m$), the system cannot reach a minimum in energy, and thus, the object will sink or float even under an applied magnetic field.

MagLev may be also understood—perhaps more intuitively—from the perspective of interacting physical forces. (Section S3 gives the quantitative descriptions.) MagLev achieves levitation of a diamagnetic object suspended in a paramagnetic medium by balancing the magnetic force and the force of gravity the object experiences. At equilibrium, these two forces are equal in magnitude but act in opposite directions. Since the physical force is the spatial derivative of energy, the statements are equivalent that the total energy of the system reaches a minimum and that the magnetic force counterbalances the gravity acting on the levitated object (and the displaced paramagnetic medium).

S2.2 What is Important? ρ, $\Delta\rho$, χ, $\Delta\chi$, $\vec{B}$, $\nabla\vec{B}$, $\vec{g}$, V, …?

The force of gravity and the buoyancy acting on any object ($\overrightarrow{F_g}$) suspended in any medium depends on three parameters: the acceleration due to gravity $\vec{g}$, the volume of the object V, and the difference in density between the object and the surrounding medium $\Delta\rho$. The magnetic force ($\overrightarrow{F_m}$, for a homogeneous diamagnetic sphere) depends on the volume of the object V, the magnitude of the magnetic field $||\vec{B}||$, the positional variation of the magnetic field (i.e., the magnetic field gradient $\nabla\vec{B}$) at the position where the object is situated in the magnetic field, and the difference in magnetic susceptibility $\Delta\chi$ of the diamagnetic object and the suspending medium that surrounds it. MagLev thus requires considerations of the physical (and also chemical) characteristics of the diamagnetic object, properties of the surrounding medium, and the strength and gradient of the magnetic field in space.

S3. The "Standard" MagLev System and its Quantitative Description

Eqs S3-6 give the quantitative relationship for the gravitational and magnetic energies and the physical forces in MagLev systems. In these equations, U_g is the gravitational energy of an object suspended in a medium under gravity. (The reference point is defined as z=0.) U_m is the magnetic energy of a diamagnetic object suspended in a paramagnetic medium under an applied magnetic field. $\overrightarrow{F_g}$ is the buoyancy-corrected gravitational force acting on the suspended object. $\overrightarrow{F_m}$ is the magnetic force the suspended diamagnetic object experiences as a result of direction interaction of the magnetic field and the paramagnetic medium that surrounds it. ρ_s is the density of the object. ρ_m is the density of the paramagnetic medium. V is volume of the object. $\vec{g}$ is the acceleration due to gravity (where $||\vec{g}||$ is 9.80665 m s^{-2} on earth). z is the z-coordinate of the object as defined in Figure 1A. χ_s is the magnetic susceptibility of the object.

χ_m is the magnetic susceptibility of the paramagnetic medium. μ_0 is the magnetic permeability of free space. $\vec{B}$ is the magnetic field. ∇ is the del operator. Eq S7 describes the conditions under which the system reaches equilibrium, and the object achieves stable levitation. Eq S8 describes the balance of the physical forces at equilibrium.

$$U_g = (\rho_s - \rho_m)Vgz \tag{S3}$$

$$U_m = -\frac{1}{2}\frac{(\chi_s - \chi_m)}{\mu_0}V\vec{B} \bullet \vec{B} \tag{S4}$$

$$\overrightarrow{F_g} = -\nabla U_g = (\rho_s - \rho_m)V\vec{g} \tag{S5}$$

$$\overrightarrow{F_m} = -\nabla U_m = \frac{(\chi_s - \chi_m)}{\mu_0}V\left(\vec{B} \bullet \nabla\right)\vec{B} \tag{S6}$$

$$\frac{d(U_g + U_m)}{dz} = 0 \tag{S7}$$

$$\overrightarrow{F_g} + \overrightarrow{F_m} = 0 \tag{S8}$$

S3.1 Relating Density to the Height of Levitation

The standard MagLev system uses an approximately linear magnetic field gradient to levitate diamagnetic objects in a paramagnetic medium. Eq S9 gives the magnetic field strength along the central axis between the two magnets (north-poles facing in this example). In eq S9, B_0 is the strength of the magnetic field at the center on the top face of the bottom magnet. d is the distance of separation between the two magnets. The origin of the MagLev frame of reference is placed at the center on the top face of the bottom magnet. At *d* = 45 mm (or below), the magnets generate an approximately linear field between the magnets along the central axis, and the field has a z-component only, B_z, because of the symmetry of the field on the x-y plane (Figure 1A).

Eq S10 gives the magnetic energy, U_m, of a suspended diamagnetic object and the equal volume of the paramagnetic medium displaced by the object under the applied magnetic field described by eq S9. Eqs S3 and S10 were used to construct the plots in Figure 1F.

$$\vec{B} = \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} \approx \begin{pmatrix} 0 \\ 0 \\ -\frac{2B_0}{d} z + B_0 \end{pmatrix} \tag{S9}$$

$$U_m = -\frac{1}{2} \frac{(\chi_s - \chi_m)}{\mu_0} V B_0^2 \left(-\frac{2}{d} z + 1\right)^2 \tag{S10}$$

Using the explicit expression of the magnetic field (eq S9), we can solve eq S7 for the z-coordinate, or the levitation height h, at which the object reaches stable levitation. Since the origin is placed on the top face of the bottom magnet, the levitation height is the distance from the centroid (the geometric center) of the object to the bottom magnet. When deriving eq S11, we assumed that the object can be quantitatively treated as an infinitesimally small volume. Rearranging eq S11 gives eqs S12-14, describing the density of the levitated object as a function of its levitation height, h.

$$h = \frac{(\rho_s - \rho_m) g \mu_0 d^2}{(\chi_s - \chi_m) 4 B_0^2} + \frac{d}{2} \tag{S11}$$

$$\rho_s = \alpha h + \beta \tag{S12}$$

$$\alpha = \frac{(\chi_s - \chi_m) 4 B_0^2}{g \mu_0 d^2} \tag{S13}$$

$$\beta = \rho_m - \frac{(\chi_s - \chi_m) 2 B_0^2}{g \mu_0 d^2} \tag{S14}$$

S3.2 Approximate and Exact Solutions for a Spherical Object

Eq S6 represents an approximation that assumes that the magnetic fields generated by the magnetized object and the magnetized medium are negligible relative to the field generated by the permanent magnets. An exact expression would include the perturbations to the magnetic field due to the magnetization of the object and the medium. A fully general and invariant expression is attainable, but is beyond the scope of this review.[8] The much simpler, and most relevant case is when the magnetic field varies slowly in space relative to the size of the sample. For a spherical object, this expression was first derived in the context of electric fields, where the effect is commonly referred to as dielectrophoresis.[9] Eq S15 is the analogous equation for the magnetic force on a spherical object suspended in a magnetic medium under the influence of an inhomogeneous, but gradually varying magnetic field.[8]

$$F_{mag}^{'} = \frac{3}{2}\mu_m \left(\frac{\mu_s - \mu_m}{2\mu_m + \mu_s}\right) V\nabla\vec{B}^2 \qquad \text{(S15)}$$

In eq S15, $\vec{B}$ represents the magnetic field generated by the magnets alone. μ_s is the magnetic permeability of the object. μ_m is the magnetic permeability of the suspending medium. The following algebraic re-arrangement enables us to compare eq S16 and eq S6 directly.

$$F = \frac{1}{2}\frac{\chi_s - \chi_m}{\mu_0}\left(\frac{1+\chi_m}{1+\frac{2}{3}\chi_m + \frac{1}{3}\chi_s}\right) V\nabla\vec{\boldsymbol{B}}^{\boldsymbol{2}} = \frac{1}{2}\frac{\chi_s - \chi_m}{\mu_0}\alpha V\nabla\vec{\boldsymbol{B}}^{\boldsymbol{2}} \qquad \text{(S16)}$$

Here we see that the rigorous approach effectively adds a correction factor α (the term in the parenthesis in eq S16). To gauge the importance of this factor, we can perform a Taylor expansion around small values of χ_m and χ_s and only keep first-order terms:

$$\alpha \approx 1 + \frac{1}{3}\chi_m - \frac{1}{3}\chi_s + \cdots \qquad \text{(S17)}$$

The dominant contribution to deviate α from unity will be from χ_m, which does not exceed 10^{-3} for aqueous paramagnetic salts ordinarily used in MagLev we describe in the Review. The correction due to magnetization of the medium will therefore be < 0.1%. A correction this small is well below the precision of measuring the position of the object, and can therefore be safely neglected. If, however, some type of non-standard medium or sample were used (e.g., a superparamagnetic fluid, such as a ferrofluid, or a (super)paramagnetic object), the levitation height would be modified by $h \rightarrow h/\alpha$. The case of a ferrofluid would include further modifications to the derivation, to consider the permanent magnetization of the fluid and/or object and magnetic hysteresis, and is beyond the scope of this review.

S4. Error Analysis

To estimate the experimental errors in measuring the unknown density of samples using the "relative" approach, we assume that the experimentally determined constants α and β (eqs S13 and S14) from the calibration curves are known exactly, and treat the uncertainty in determining the levitation height is the only source of error. Eq S18 gives the equation to calculate the associated experimental error.

$$\delta\rho_s = \left|\frac{d\rho_s}{dh}\right|\delta h = |\alpha|\delta h \qquad \text{(S18)}$$

The second approach to measure an unknown density of a sample is to directly calculate its value from its levitation height using known values of the physical parameters described in eq S12, including the density of the medium ρ_m, the magnetic susceptibilities of the sample x_s, and the medium x_m, the magnitude of the magnetic field B_0, the distance of separation between the two magnets d, and the levitation height h. This “direct” approach does not require the use of density standards to calibrate the system; it, however, places three requirements on the users: (i) a working knowledge of the physical principles of the system; (ii) known values for all the physical parameters described in eq S12 at the time of density measurement; and (iii) considerations of environmental influences on the measurements, including the temperature-dependency of ρ_m, x_m, x_s, and B_0. Typical experimental values for the standard MagLev system are given elsewhere in detail.[10]

The error analysis for the “direct” approach is more complex than the “relative” approach because of the need to account for every source of random error when using eq S12 to calculate the unknown density of the sample. The density of the sample ρ_s can be treated as a function of the following six independent variables, including the magnitude of the magnetic field B_0, the magnetic susceptibility of the sample x_s, the concentration of the paramagnetic medium c, the distance of separation between the magnets d, the levitation height h, and the ambient temperature T. (The density of the paramagnetic medium and the magnetic susceptibility of the medium are not independent variables in that both parameters are a function of the concentration of the paramagnetic medium and the ambient temperature.) Eq S19 gives the standard expression to calculate the error in the density of the sample $\delta\rho_s$ when directly using eq S12 to estimate ρ_s. Example of error analysis for the “direct” approach is given in detail elsewhere.[10] For the majority of the density measurements, the “relative” approach is almost always preferred due to its simplicity and ease with which to implement experimentally.

$$\delta\rho_s = \sqrt{\left(\frac{\partial\rho_s}{\partial T}\delta T\right)^2 + \left(\frac{\partial\rho_s}{\partial c}\delta c\right)^2 + \left(\frac{\partial\rho_s}{\partial x_s}\delta x_s\right)^2 + \left(\frac{\partial\rho_s}{\partial h}\delta h\right)^2 + \left(\frac{\partial\rho_s}{\partial d}\delta d\right)^2 + \left(\frac{\partial\rho_s}{\partial B_0}\delta B_0\right)^2} \quad \text{(S19)}$$

S5. Theoretical Guide to Adjust Sensitivity and Dynamic Range

We define the sensitivity of a MagLev system as the change in levitation height per unit change in density—i.e., the slopes of the calibration curves on plots of levitation height vs. density (Figure 5B). We define the dynamic range as the range of density over which we can perform density measurements. Operationally, the dynamic range of the standard MagLev system spans the entire distance of the separation between the two magnets (i.e., the entire range of the linear magnetic field). Dynamic ranges for MagLev systems other than the standard configuration may be extended to include the nonlinear portions of the magnetic field. This review primarily focuses on the approaches that exploit approximately linear magnetic fields.

Eqs S20 and S21 give the quantitative description of the sensitivity and dynamic range of density measurements for the standard MagLev system. Eq S22 describes the density of the paramagnetic medium as a function of the ambient temperature T, the type of solvent used to prepare the medium, and the concentrations of dissolved species, both diamagnetic and paramagnetic). In eq S22, c_i $(i = 1, 2, \ldots)$ stands for the concentration of the solute i.

$$S_z = \frac{\Delta h}{\Delta \rho} = \frac{\mu_0 g}{(\chi_s - \chi_m)\left(\frac{2B_0}{d}\right)^2} \quad \text{(S20)}$$

$$\Delta\rho_{range} = \rho_{z=0} - \rho_{z=d} = \frac{2(\chi_s - \chi_m)}{\mu_0 g}\left(\frac{2B_0}{d}\right)B_0 \quad \text{(S21)}$$

$$\rho_m = f(T, solvent, c_1, c_2, \ldots) \quad \text{(S22)}$$

In eq S20, the sensitivity of the MagLev system S_z (i.e., the slope of a calibration curve), is expressed as the ratio of the change in levitation height Δh to the change in density $\Delta\rho$. The quantity $2B_0/d$ is the gradient of the linear magnetic field between the two magnets. The dynamic range is the difference in density for objects that levitate between the two magnets—that is, the range in density that can be levitated using the entire linear gradient between the magnets. The middle point of the dynamic range is the density of the paramagnetic medium ρ_m. These three equations form the theoretical basis used to guide the experimental design to tune the sensitivity and the dynamic range of the standard MagLev system, and can be, in fact, extended to any MagLev system using a linear magnetic field so long as the linear field (i) is aligned with the vector of gravity, and (ii) has its null point (where B = 0 T) located physically in the midpoint of the gradient. These equations also show that these two analytical parameters—sensitivity and dynamic range—are inherently coupled and trade-offs often need to be made.

We emphasize that precision and accuracy—two closely related but distinct characteristics of any analytical system—are also relevant to the discussions of the sensitivity and dynamic range. Precision describes the reproducibility of the measurements—that is how reproducible the measurements are. Accuracy describes the "closeness" of a measured value to the true value of the sample (e.g., the true density of a sample). A measurement (e.g., of a sample in a MagLev system) can be precise but not accurate if the system is not calibrated correctly. Both precise and accurate density measurements can be achieved using MagLev systems optimized for high-sensitivity measurements; for such measurements, high-quality density standards are essential to calibrate the system.

S6. Understanding and Controlling Orientation of Levitated Objects

For an arbitrarily-shaped, homogenous object, the total magnetic potential energy of the object can be described by eq S23, where $\beta = 2{B_0}^2/\mu_0 d^2$, B_0 is the field at the surface of one of the magnets, and *d* is the distance between the faces of the magnets. In this equation, we assumed that the magnetic field **B** is linear, and $\Delta\chi$ is uniform throughout the volume of the object.

$$U_{mag} = \int_V u_{mag} dV = \int_V -\frac{\Delta\chi}{2\mu_0} |\mathbf{B}(\mathbf{r})|^2 dV = \beta\Delta\chi \int_V z^2 dV \qquad \text{(S23)}$$

To calculate the integral term, we must parameterize in terms of the body-fixed, local coordinate system of the object $\mathbf{r}' = [x', y', z']$. In general, a principal coordinate system can be found that coincides with the geometric centroid of the object. In this coordinate system, the determining factors in describing the orientation of the object are the second-moments of area $\lambda_{k'}$ for $k' \in \{x', y', z'\}$, as defined in eq S24.

$$\lambda_{k'}{}^2 = \frac{1}{V} \int_V k'^2 \, dV' \qquad \text{(S24)}$$

The orientation of an arbitrary homogenous object can be described entirely by the competition between the $\lambda_{k'}$ values. In particular, for a linearly varying magnetic field, the principal axis associated with the smallest of the $\lambda_{k'}$ values always aligns with the z-axis of the MagLev device.

To understand this behavior intuitively, we consider the example of a cylinder (Figure 10B), which has principal axes that have a double degeneracy. If we choose a principal coordinate system such that the z'-axis aligns with the shaft, then $\lambda_{x'} = \lambda_{y'}$. Finally, we consider rotation about the x-axis (this is general, because of the degeneracy), and so only $R = (\lambda_{z'}/\lambda_{y'})^2$, the competition between $\lambda_{y'}$ and $\lambda_{z'}$, matters. In this case, the magnetic potential energy reduces to eq S25.

$$U(\alpha, h) = \beta V \Delta\chi \lambda_{y'}{}^2 (1 - R) \sin^2\alpha + \Delta\chi V \beta V h^2 \qquad \text{(S25)}$$

Inspection of eq S25 reveals that the magnetic torque and height of the object are decoupled; this behavior generalizes to non-degenerate objects as well, and enables us to separately find the equilibrium height of the centroid of the object and the orientation of object around the principal axes. Figure 10B shows a plot of the first term of eq S25 (the angle dependent part). If $R > 1$, the shaft is longer than the face is wide, the minima in energy occur $\alpha \in \{90^\circ, 270^\circ\}$, and the z'-axes orients perpendicular to the z-axis (shaft pinned to the x/y plane). If $R < 1$, the shaft is shorter than the face is wide, the minima in energy occur $\alpha \in \{0^\circ, 180^\circ\}$, and the z'-axes orients parallel to the z-axis (shaft pinned to the z-axis). In all cases, the principal axis associated with the smallest second-moment of area orients along the magnetic field gradient (z-axis). A transition between the behaviors occurs at R = 1; this behavior can be seen for a variety of objects with the same type of degeneracy in Figure 10C.

S7. Quality Control: Heterogeneity in Density in Injection-Molded Parts

For a non-spherical object with a density-based defect, we first analyze it based on shape and move to the principal axis where the origin of body-fixed coordinate system of the object lies at the geometric centroid, and the axes are aligned with the principal axes. If we consider the simple example of rectangular rod (Figure 11A) with length L, width W, and density ρ_r, together with a cubic inclusion with density ρ_i and volume V_i, located at a distance w_i from the centroid of the rod, and constrained to the w-axis of the rod, then the angle-dependent part of potential energy reduces to eq S26.

$$U(\theta) = \frac{1}{12}\beta \Delta \chi V(L^2 - W^2)\sin^2\theta + (\rho_i - \rho_r)V_i g w_i \cos\theta \qquad \text{(S26)}$$

There are two components to the rotational potential energy on the object, the first due to the shape of the object (Section S6), and a new term for the projection of the center-of-mass of the object along the z-axis of the MagLev device. In general, the second term perturbs solutions from the first term, and solutions will take the form of $\theta = \theta_{mag} + \alpha$, where θ_{mag} is the equilibrium orientation of the object due to shape alone, and α the added tipping due to the inclusion. If $\rho_i < \rho_r$, the inclusion is less dense than the object (e.g., air) and the side of the object with the inclusion will tend to tip up. If $\rho_i > \rho_r$, the inclusion is more dense than the object, and the side of the object with the inclusion will tend to tip down. For example, Figure 11B shows the theoretical and experimental levitation angles α for 3D-printed rods having a known type of inclusions that vary in size at the same position.

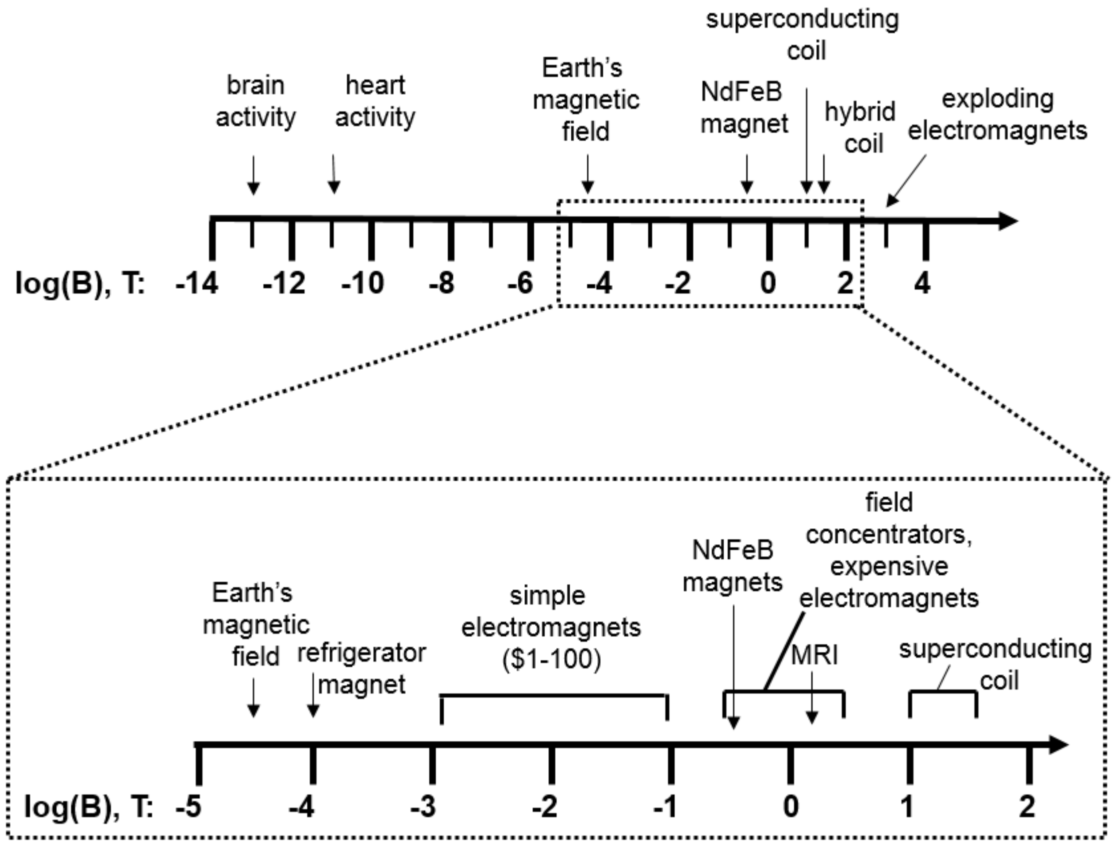


Figure S1. Experimentally accessible range of magnetic fields. We constructed this plot using data from various sources.[10–13] T is tesla.

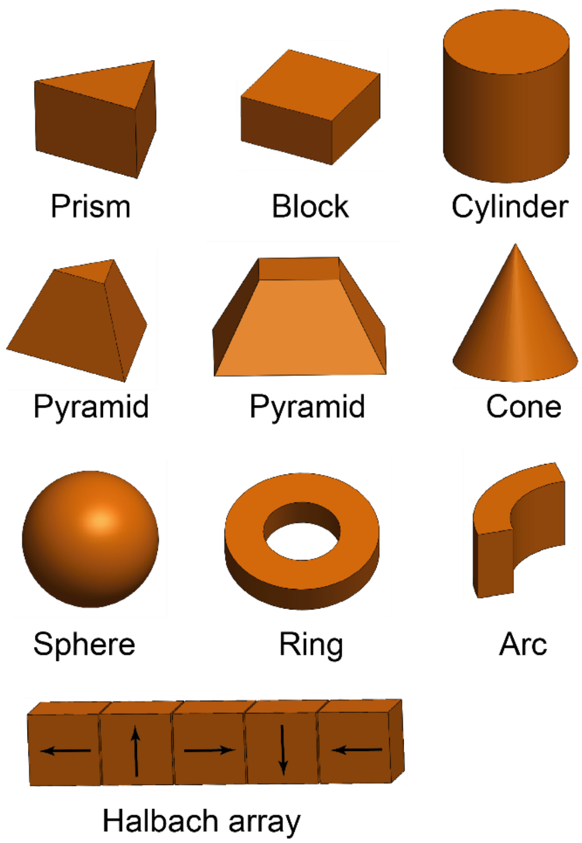
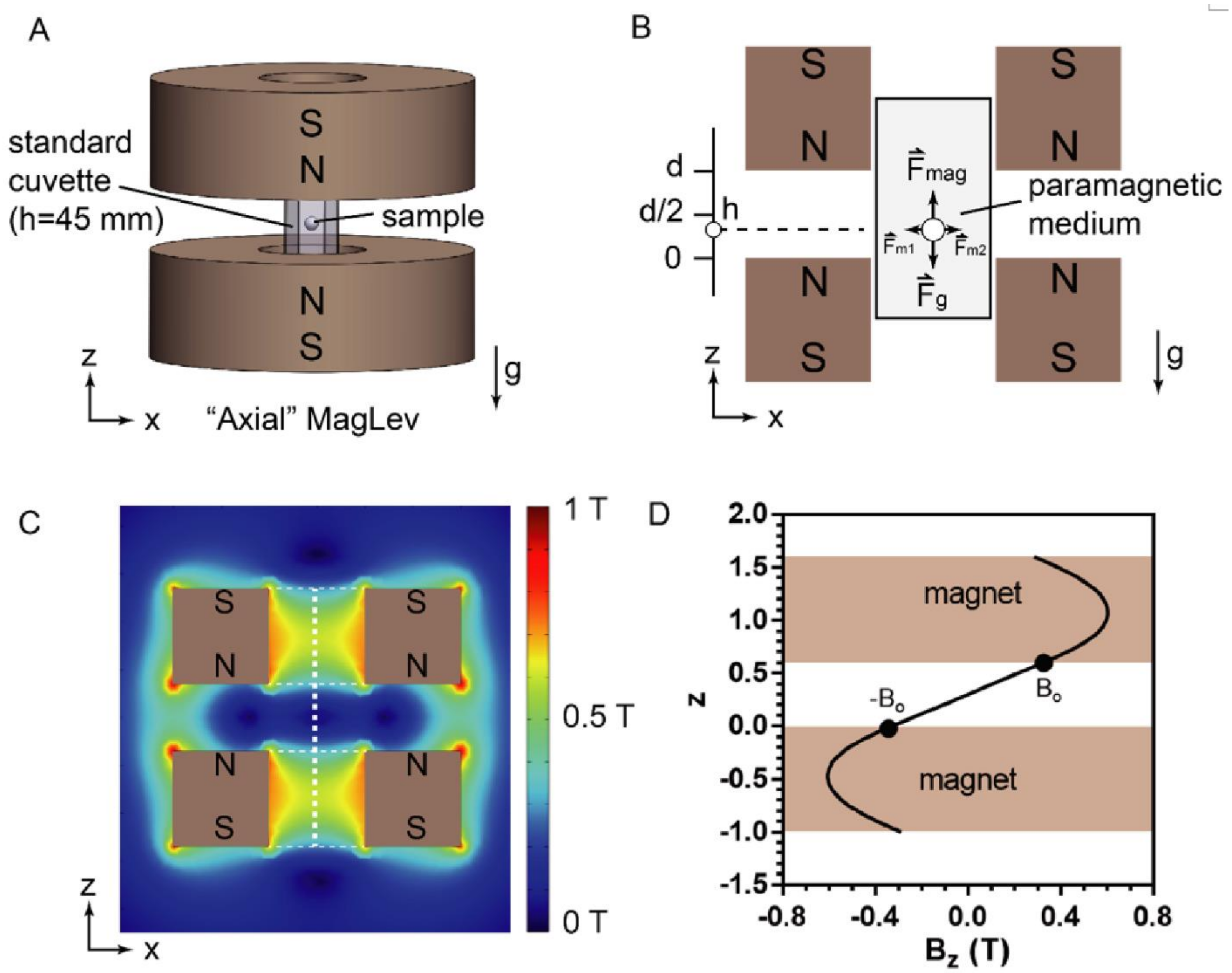
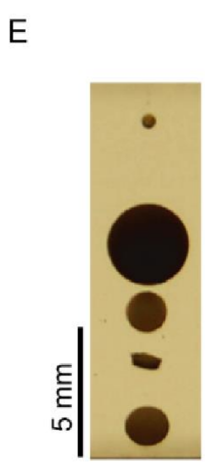


Figure S2. Different shapes of commercial NdFeB permanent magnets. The black arrows in the Halbach array indicate the direction of the magnetization of the cube magnets, and the magnetic field underneath the array is stronger than the field above it. NdFeB magnets may be obtained from different vendors (for example, kjmagnetics.com, magnet4less.com, and supermagnetman.com).





	ρ (g/cm3)	
	measured	reported
air	−0.01±0.02	0.001
Torlon®	1.37±0.02	1.40
Teflon®	2.11±0.01	2.15
aluminum	2.68±0.02	2.74
zirconium silicate	3.70±0.05	3.73

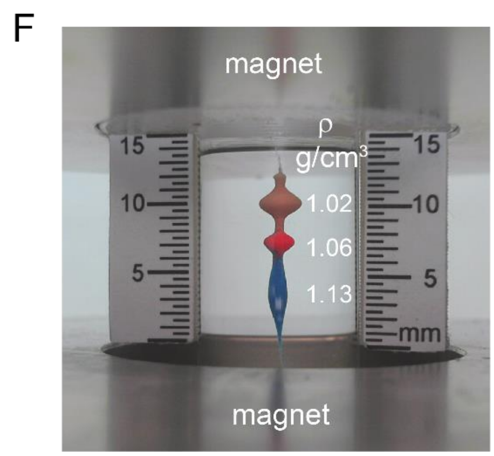


Figure S3. Axial MagLev (A, B) Schematic of an “axial” MagLev device (with north-poles-facing) and a standard cuvette (45 mm in height) containing a paramagnetic medium and a sample. (C) The magnetic field visualized using Comsol simulation. (D) The strength of the magnetic field along the central axis of the “axial” MagLev device. (E) Density measurement of five materials levitated simultaneously in a solution of 3.0 M $DyCl_3$. (F) Three types of particles with different densities (all ~40 μm in diameter) suspended in an aqueous solution of 0.5 M $MnCl_2$ were focused axially in the MagLev device, and separated into three populations. Within each population, the distribution of the particles along the z-axis represents the heterogeneity in density of these particles. Source: Images (A-E).[14]

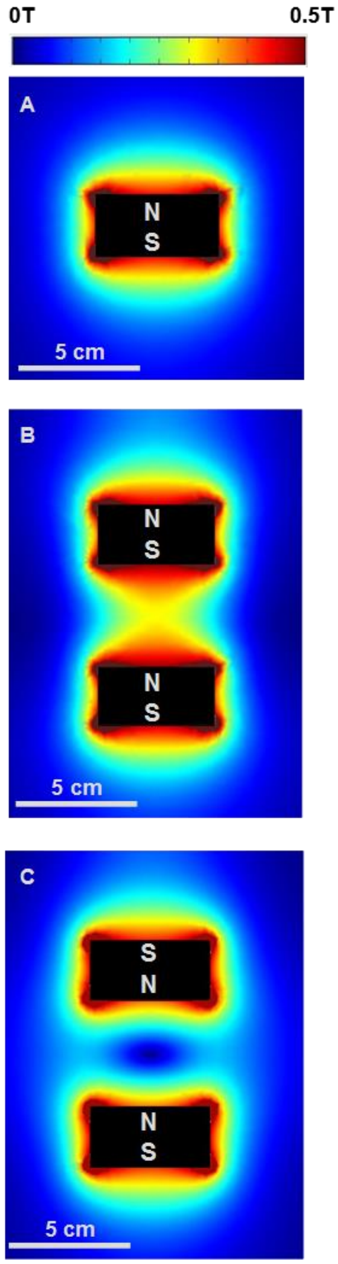


Figure S4. Visualization of the magnetic fields surrounding magnets using numerical simulations in COMSOL®. Three exemplary arrangements of NdFeB magnets (25 mm × 50 mm × 50 mm) include (A) a single magnet, B) opposite-poles facing configuration, C) like-poles facing configuration.

Table S1. Common materials as density standards

Water-insoluble materials as density standards	Density[a] (g/cm^3)
Polymers	
high-density polyethylene	0.97
polydimethylsiloxane (PDMS)	1.04
polystyrene	1.05
poly(styrene-co-acrylonitrile)	1.08
poly(styrene-co-methyl	1.14
nylon 6/6	1.14
poly(methyl methacrylate)	1.18
polycarbonate	1.22
neoprene rubber	1.23
polyethylene terephthalate (Mylar)	1.40
polyvinylchloride (PVC)	1.40
cellulose acetate	1.42
polyoxymethylene (Delrin)	1.43
polyvinylidene chloride (PVC)	1.70
polytetrafluoroethylene (Teflon)	2.20
Organic liquids	
toluene	0.865
1,2,3,4-tetramethylbenzene	0.905
methyl methacrylate	0.936
4,N,N-trimethylaniline	0.937
4-methylanisole	0.969
anisole	0.993
3-fluorotoluene	0.997
2-fluorotoluene	1.004

fluorobenzene	1.025
3-chlorotoluene	1.072
chlorobenzene	1.107
2,4-difluorotoluene	1.120
2-nitrotoluene	1.163
nitrobenzene	1.196
1-chloro-2-fluorobenzene	1.244
1,3-dichlorobenzene	1.288
1,2-dichlorobenzene	1.305
dichloromethane	1.325
3-bromotoluene	1.410
bromobenzene	1.491
chloroform	1.492
1,1,2-trichlorotrifluoroethane	1.575
1-bromo-4-fluorobenzene	1.593
hexafluorobenzene	1.612
carbon tetrachloride	1.630
tetradecafluorohexane	1.669
2,5-dibromotoluene	1.895
perfluoro(methyldecalin)	1.950
1,2-dibromoethane	2.180
iodomethane	2.280
dibromomethane	2.477
tribromomethane	2.891

[a] The values of densities are obtained from sigma.com and reference[15].

References

[1] R. P. Feynman, R. B. Leighton, M. Sands, *The Feynman Lectures on Physics*, Basic Books, New York, NY, **2010**; Volume II, 16-5

[2] R. Boča, *Theoretical Foundations of Molecular Magnetism*, Elsevier, **1999**; Volume I, 315–344.

[3] J. F. Herbst, J. J. Croat, *J. Magn. Magn. Mater.* **1991**, *100*, 57–78.

[4] J. F. Herbst, J. J. Croat, F. E. Pinkerton, W. B. Yelon, *Phys. Rev. B* **1984**, *29*, 4176–4178.

[5] S. Odenbach, *Magn. Electr. Sep.* **1998**, *9*, 1–25.

[6] A. K. Gupta, M. Gupta, *Biomaterials* **2005**, 3995–4021.

[7] S. Laurent, D. Forge, M. Port, A. Roch, C. Robic, L. Vander Elst, R. N. Muller, *Chem. Rev.* **2008**, 2064–2110.

[8] C. Rinaldi, *Chem. Eng. Commun.* **2010**, *197*, 92–111.

[9] V. Denner, H. A. Pohl, *J. Electrostat.* **1982**, *13*, 167–174.

[10] K. A. Mirica, S. S. Shevkoplyas, S. T. Phillips, M. Gupta, G. M. Whitesides, *J. Am. Chem. Soc.* **2009**, *131*, 10049–10058.

[11] D. Nakamura, A. Ikeda, H. Sawabe, Y. H. Matsuda, S. Takeyama, *Rev. Sci. Instrum.* **2018**, *89*, 095106.

[12] D. Cohen, *Science.* **1968**, *161*, 784–786.

[13] P. G. Anastasiadis, P. Anninos, A. Kotini, N. Koutlaki, A. Garas, G. Galazios, *Clin. Exp.*

Obstet. Gynecol. **2001**, 28, 257–260

[14] S. Ge, G. M. Whitesides, *Anal. Chem.* **2018**, *90*, 12239–12245.

[15] D. Mackay, W. Y. Shiu, K. Ma, S. C. Lee, *Properties and Environmental Fate Second Edition Introduction and Hydrocarbons*, Taylor & Francis Group, **2006**.