## SUPPLEMENTARY INFORMATION

# The Magnitude of Lift Forces Acting on Drops and Bubbles in Liquids Flowing inside Microchannels

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**Abstract.** This supplementary information document contains: (1) additional information about the fluids used in this study; (2) a table with the characteristics of the lift forces for all pairs of fluids investigated here; and (3) graphs, for each system investigated in microchannels, showing the comparison between experimental measurements and the predictions of deformation-induced and empirical formulas for the lift force. A separate Excel file contains the hydrodynamic parameters of all experiments used for Fig. 3 in the main paper.

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## 1. Further information about the fluids used in this study

Table ST1 shows a brief description of the fluids we used, including their chemical or trade name, the abbreviation we used in the paper, their chemical purity (if applicable), and their source or supplier.

With the exception of water and nitrogen gas (whose properties are accurately tabulated in the literature), and of RT500 (whose properties were supplied by the manufacturer), we determined experimentally the fluids' density, viscosity, and surface tension (against the relevant second phase) as a function of temperature. The supplementary information of our previous work<sup>1</sup> contains detailed information on the hydrodynamic properties of PFPHP and DySF.

Abbreviation	Chemical or trade name, and notes		
PFPHP	Perfluoroperhydrophenanthrene, 87.5% purity, F2 Chemicals Ltd.		
PFMD	Perfluoromethyldecalin, 98% purity, F2 Chemicals LTD.		
THPFO	1H,1H,2H,2H-perfluorooctanol, 97% purity, Sigma-Aldrich USA.		
DySF	Dynalene SF, Dynalene Inc. Blend of aliphatic hydrocarbons engineered for high-temperature heat transfer.		
RT500	RT500 silicone oil, Cannon Instrument Co. Viscosity standard liquid, prepared to have a dynamic viscosity of 500 mPas·s at room temperature.		
Water	Purified water, from a Milli-Q purification system.		
CsCl solution	Aqueous cesium chloride solution in various concentrations, prepared to provide a drop liquid with tunable density ( $\rho = 1120-1780 \text{ kg/m}^3$ )		
N <sub>2</sub>	Lab-supplied dry nitrogen, obtained from vaporization of liquid nitrogen.		

Table ST1. Fluids used in our study.

#### 2. Types of hydrodynamic lift in the systems we investigated

Table ST2 lists the characteristic of lift forces for all systems we reported, and the value of the lift coefficient  $C_L$  (determined by fitting) where applicable. Most of the systems exhibited a lift force consistent with either a deformation-induced lift force (eqn (1) in the paper) or with the empirical formula we proposed (eqn (11) in the paper). In a few cases, the dependence of the lift force on hydrodynamic parameters was most consistent with a deformation mechanism but the lift force was bigger by a large numerical factor; we labeled this type of lift as 'deformation-proportional'.

In our survey, we did not characterize with equal accuracy every system; some of our early measurements were probably affected by larger errors than those performed after we optimized our experimental techniques. While Table 1 in the main paper lists the systems that we investigated most carefully, Table ST2 lists all data that is shown in Figure 3 in the main paper. In a few cases, the range over which we varied the capillary number was too limited to discern the lift type, and we labeled them as 'not enough  $Ca_P$  range' in Table ST2. Lastly, we have also encountered cases in which the dependence of the lift force on the hydrodynamic parameters was too complex to identify it with a deformation-induced or empirical lift mechanism, and we labeled them 'mixed'. These data sets, although less instructive by themselves, fit in the overall trend which is visible in Figure 3 in the main paper.

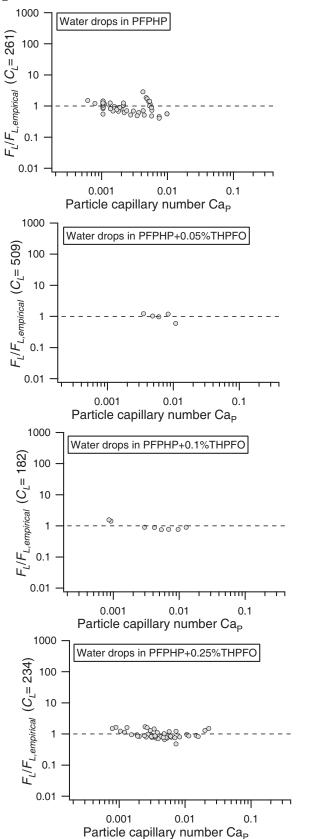
#### **3.** Additional data for individual systems

Figures S1–S5 segregate the data sets that have been grouped together in Figures 3a, 3b, and 5c in the main paper. For each individual data set we have plotted the ratios of the measured lift force  $F_L$  to (i) the empirical lift force  $F_{L,empirical}$  (with the  $C_L$  specific to the data set) and to (ii) the deformation-induced lift force  $F_{L,deformation}$ .

The order of graphs is the same as the order of systems in Table ST2. The channel size (or sizes) used, if not specified in the graph, was  $125(W) \times 200(H) \mu m$ .

<b>Table ST2.</b> Lift force type, lift coefficients $C_L$ , and correction factors for the deformation lift
formula for all data reported in the paper.

System	Lift type	Lift coefficient $C_L$	Correction for deformation lift
Water drops in PFPHP	Empirical	261	-
Water drops in PFPHP + 0.05%THPFO	Empirical	509	-
Water drops in PFPHP + 0.1%THPFO	Empirical	182	-
Water drops in PFPHP + 0.25%THPFO	Empirical	234	-
Water drops in PFPHP + 1%THPFO	Empirical	370	-
Aq. CsCl solution (1120–1780 kg/m <sup>3</sup> ) in PFPHP + 1%THPFO	Empirical	289	-
Water drops in PFPHP + 2%THPFO	Empirical	636	-
Water drops in PFPHP + 5%THPFO	Empirical	1278	-
Water drops in PFPHP + 10%THPFO	Empirical	1230	-
Water drops in PFPHP + 20%THPFO	Proportional to deformation	1271	229
Water drops in THPFO	Empirical	1026	-
N <sub>2</sub> bubbles in PFPHP + 0.5%THPFO	Mixed	300	8.7
N <sub>2</sub> bubbles in PFPHP + 5%THPFO	Mixed	254	6.5
Water drops in PFMD + 2%THPFO	Empirical	1250	-
Water drops in PFMD + 5%THPFO	Not enough Ca <sub>P</sub> range	446	339
Water drops in PFMD + 10%THPFO	Empirical	515	-
N <sub>2</sub> bubbles in PFMD + 2%THPFO	Mixed	116	3.6
N <sub>2</sub> bubbles in PFMD + 10%THPFO	Not enough Ca <sub>P</sub> range	365	9.6
Aqueous CsCl solution (1520 kg/m <sup>3</sup> ) in DySF	Proportional to deformation	581	97
Nitrogen bubbles in DySF	Proportional to deformation	315	35
Nitrogen bubbles in RT500	Deformation	-	2.4



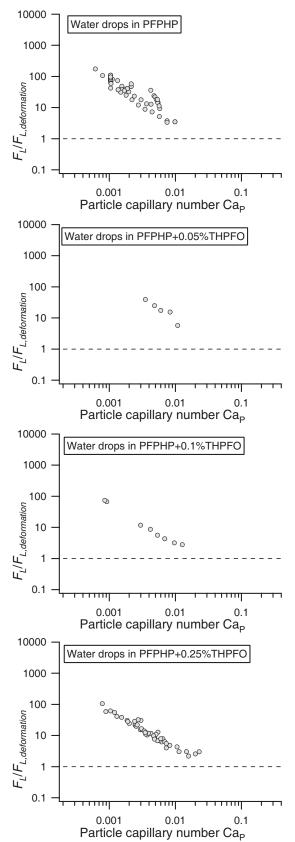


Figure S1. Additional lift force data.

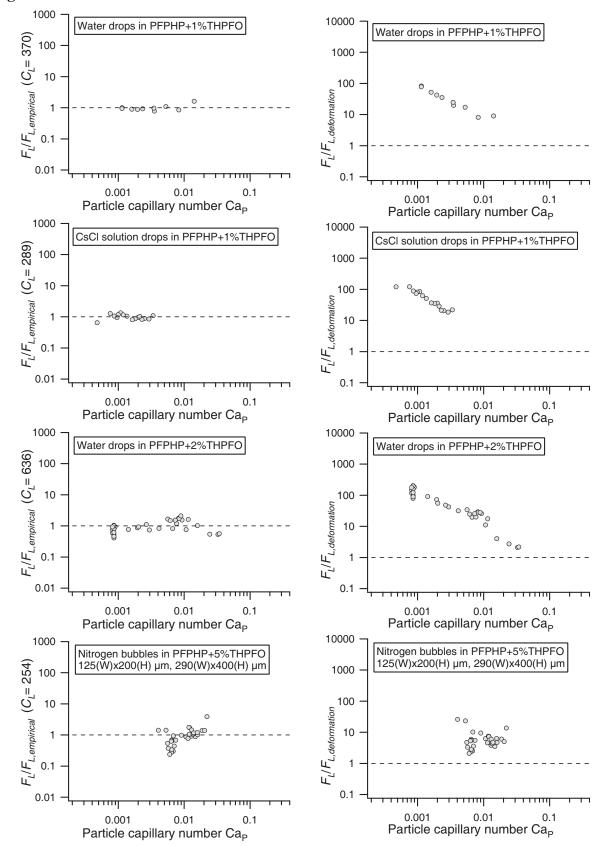


Figure S2. Additional lift force data.

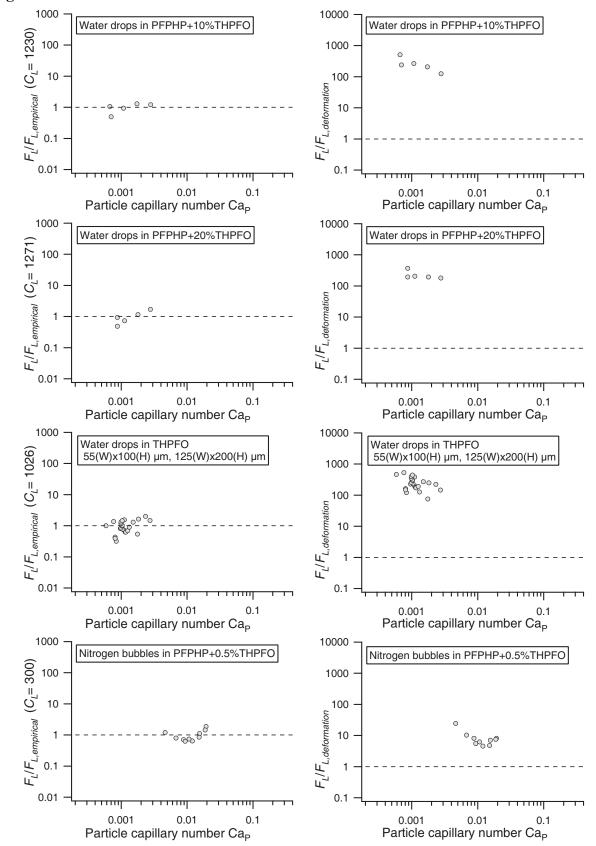
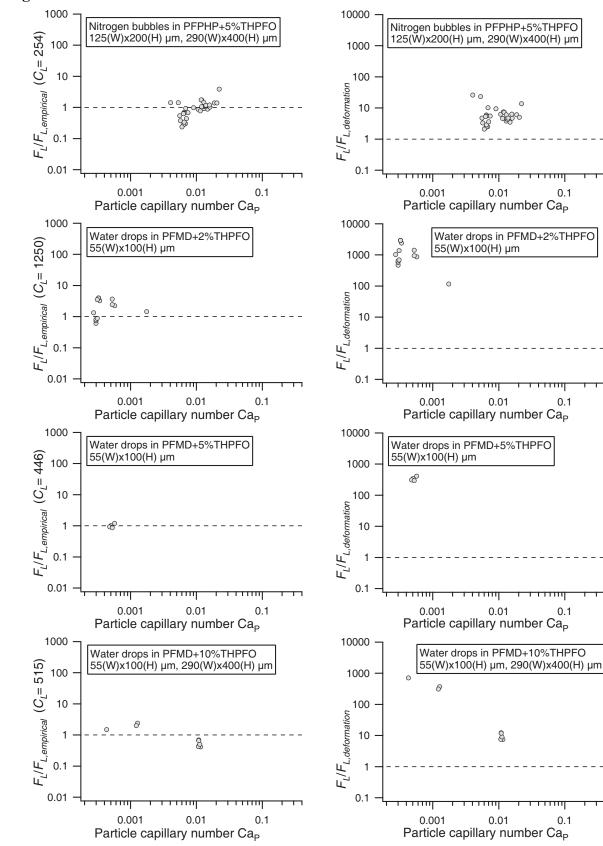


Figure S3. Additional lift force data.



0.1

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Figure S4. Additional lift force data.

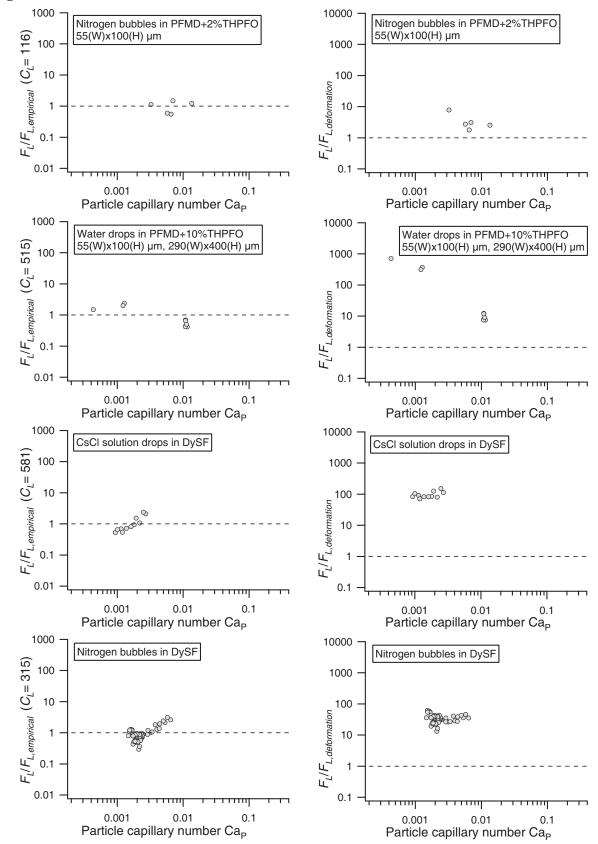


Figure S5. Additional lift force data.

## 4. Hydrodynamic parameters of all experiments used for Fig. 3

The separate Excel spreadsheet file included in the ESI lists the hydrodynamic parameters of all experiments used to calculate the data shown in Fig. 3 in the main paper: the carrier and drop fluids, H,  $\alpha$ , r, d,  $V_{avg}$ ,  $\mu_C$ ,  $\mu_{drop}$ ,  $\gamma$ ,  $\rho_C$ ,  $\rho_{drop}$ , Re<sub>C</sub>, Re<sub>P</sub>, Ca<sub>P</sub>, and  $F_L$ . The first spreadsheet contains numeric codes we assigned to each of the continuous and dispersed phase fluids, and the second spreadsheet contains the experimental parameters. Experiments with the same H and continuous and dispersed phase fluids have been grouped together; within each such group the experiments have been sorted from the lowest to the highest capillary number.

### References

 C. A. Stan, L. Guglielmini, A. K. Ellerbee, D. Caviezel, H. A. Stone and G. M. Whitesides, *Phys. Rev. E*, 2011, **84**, 036302.