Supporting Information

A Transparent Membrane for Active Noise Cancellation

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Calculation of the Acoustic Waves from the Measured Pressures

In both sections of the impedance tube plane waves travel in positive and negative x-direction (Figure S1). These waves can be decomposed into their frequency components with radial frequency ω and wavenumber k ($k = \omega / c$, where c is the speed of sound). At each location x the pressure is the sum of the right and left traveling waves:

$$p_{\mathrm{F}} = F_{\mathrm{R}} e^{i(\omega t - kx)} + F_{\mathrm{L}} e^{i(\omega t + kx)}, \tag{S1}$$

$$p_{\mathrm{B}} = B_{\mathrm{R}} e^{i(\omega t - kx)} + B_{\mathrm{L}} e^{i(\omega t + kx)}, \tag{S2}$$

Microphones measure the pressures at four locations (x_{j}, j = 1 - 4) as a function of time. Short term Fourier transformation can decompose the measured pressure signals of the microphones into their frequency components (p_{j}, j = 1 - 4). With Equation S1, and S2 one can calculate the (complex) amplitudes of the traveling waves in both sections of the tube:[1,2]

$$F_{\mathrm{R}} = \frac{i\left(p_1 e^{ikx_2} - p_2 e^{ikx_1}\right)}{2\sin\left(k\left(x_1 - x_2\right)\right)}, \tag{S3}$$

$$F_{\mathrm{L}} = \frac{i\left(p_2 e^{-ikx_1} - p_1 e^{-ikx_2}\right)}{2\sin\left(k\left(x_1 - x_2\right)\right)}, \tag{S4}$$

$$B_{\mathrm{R}} = \frac{i\left(p_3 e^{ikx_4} - p_4 e^{ikx_3}\right)}{2\sin\left(k\left(x_3 - x_4\right)\right)}, \tag{S5}$$

$$B_{\mathrm{L}} = \frac{i\left(p_4 e^{-ikx_3} - p_3 e^{-ikx_4}\right)}{2\sin\left(k\left(x_3 - x_4\right)\right)}, \tag{S6}$$

Derivation of the Transfer Matrix

The classical transfer matrix does not directly relate the pressures of the incident and reflected waves on the left (x = 0) and right ($x = d$) sides of a sample, but the total pressures (p_0, p_{d}), and velocities (v_0, v_{d}) at the two interfaces:[1]

$$\begin{pmatrix} p_0 \\ v_0 \end{pmatrix} = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix} \begin{pmatrix} p_d \\ v_d \end{pmatrix}, \tag{S7}$$

$$p_0 = F_\mathrm{R} + F_\mathrm{L}, \tag{S8}$$

$$p_\mathrm{d} = B_\mathrm{R} e^{-ikd} + B_\mathrm{L} e^{-kd}, \tag{S9}$$

$$v_0 = \frac{F_\mathrm{R} - F_\mathrm{L}}{\rho_0 c}, \tag{S10}$$

$$v_\mathrm{d} = \frac{B_\mathrm{R} e^{-ikd} - B_\mathrm{L} e^{ikd}}{\rho_0 c}, \tag{S11}$$

In Equation S9 and S11, d is the thickness of the sample. It is not straightforward to define d for the transparent membrane, because it is thicker in the region covered by the ionic electrodes (~ 2 mm) than in the region that is not covered (~ 0.1 mm). Because the smallest wavelength in our experiments (λ ~ 0.34 m at f = 1000 Hz) is much larger than the maximum thickness of the loudspeaker ($d \approx$ 2 mm), we neglect the exponential factors in Equation S9 and S11 (the error in phase angle due to this simplification is ~ 2º). With this simplification we derive the relationship between, T_{ij} and Q_{ij}:

$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} Q_{11} + \rho_0 c Q_{21} + \frac{Q_{12}}{\rho_0 c} + Q_{22} & Q_{11} + \rho_0 c Q_{21} - \frac{Q_{12}}{\rho_0 c} - Q_{22} \\ Q_{11} - \rho_0 c Q_{21} + \frac{Q_{12}}{\rho_0 c} - Q_{22} & Q_{11} - \rho_0 c Q_{21} - \frac{Q_{12}}{\rho_0 c} + Q_{22} \end{pmatrix}, \tag{S12}$$

For a symmetric, reciprocal sample Q_{ij} has only two independent components:[1]

$$Q_{11} = Q_{22}, \tag{S13}$$

$$Q_{11}Q_{22} - Q_{12}Q_{21} = 1, \tag{S14}$$

By plugging Equation S13 and S14 into Equation S12 we obtained the analogous relationships for the components of T_{ij}:

$$T_{21} = -T_{12}, \tag{S15}$$

$$T_{11}T_{22} - T_{12}T_{21} = 1, \tag{S16}$$

Characterization of the Acoustic Properties of the Transparent Membrane

At a critical voltage, a subharmonic appears in the acoustic spectrum of the transparent membrane, which indicates the end of the linear regime (Figure S2). Figure S3 shows that the acoustic spectrum of the left traveling wave in the front section of the impedance tube (F_L) does not have a subharmonic during a frequency sweep on the transparent membrane from 150 Hz to 1000 Hz at amplitude V_a = 645 V, and bias voltage V_b = 9.0 kV. Figure S4 shows the fundamental components of the right traveling, and left traveling waves in both sections of the impedance tube during the characterization of the transparent membrane.

Mode of Oscillation of the Transparent Membrane

The Maxwell stress Equation 1 causes a reduction in thickness of the dielectric membrane. A sinusoidal excitation voltage on the membrane causes a periodic change in thickness. In this oscillation mode, the air at the two interfaces between the membrane and the air moves in opposite directions ($v_0 = -v_d$). From Equation S10 and S11 follows for an anechoic tube ($F_R = B_L = 0$) that $\hat{F}_L / \hat{B}_R = 1$ (*i.e.*, the phase angle of $\hat{F}_L / \hat{B}_R$ is 0º). When the membrane oscillates out-of-plane like a drumhead, the velocities at the two interfaces are equal ($v_0 = v_d$). From Equation S10, and S11 follows for an anechoic tube that $\hat{F}_L / \hat{B}_R = -1$(*i.e.*, the phase angle of $\hat{F}_L / \hat{B}_R$ is 180º). It is also possible, that both oscillation modes contribute equally to sound generation. In this case the phase angle of $\hat{F}_L / \hat{B}_R$ is neither 0º nor 180º. In our experiments, the phase angle was approximately 180º (Figure 4), so we conclude that the dominant oscillation mode is out-of-plane.

Active Noise Cancellation Experiment

The incident sound wave on the transparent membrane F_R is composed of the sound generated by the coil loudspeaker, and the reflection of F_L at the coil loudspeaker. The sound wave F_L is therefore modified by exciting the membrane (Equation 4). During the noise cancellation experiment, we adjust the excitation signal on the coil loudspeaker to keep the amplitude and phase of F_R constant. The amplitude of the sound generated by the coil loudspeaker is linearly proportional to the excitation amplitude, and the wave reflected from the coil loudspeaker is proportional to the incident sound wave

$$F_{\mathrm{R}} = S_{\mathrm{L}} V_{\mathrm{L}} + T_{\mathrm{L}} F_{\mathrm{L}} , \tag{S17}$$

This equation is equivalent to Equation 4 for the transparent membrane. The parameters S_L, and T_L can be determined from the same data that we used to determine the properties of the transparent membrane.

Assuming that the membrane cancels a prescribed incident noise F_R perfectly (*i.e.*, $B_R = B_L = 0$), we combine Equation 4, 7 and S17 to obtain the excitation signal on the coil loudspeaker to keep F_R constant:

$$V_{\mathrm{L}} = \left(1 - \frac{T_L S_2}{S_1}\right) \frac{F_R}{S_L} , \tag{S18}$$

With Equation 7 we calculated the excitation signal for the transparent membrane to cancel a noise wave, which consists of a sinusoidal frequency sweep from 150 Hz, and 1000 Hz of 80 dB amplitude. To ensure linearity of the membrane, we capped the amplitude of the excitation signal at 645 V. Using the calculated excitation signal for the transparent membrane, and Equation S18 we calculated the excitation signal for the coil loudspeaker to generate a constant incident acoustic wave of amplitude 80 dB for all frequencies (Figure S5a and S5b). Because the membrane did not completely cancel F_R (an assumption in the derivation of Equation S18), and because of inaccuracies in the parameters in Equation S18, F_R deviated slightly from the desired value during the experiment (Figure S5c and S5d).

Figure S6 shows the measured amplitudes of the acoustic spectrum during the noise cancellation experiment, and Figure S7 a comparison of the sound transmission loss calculated with Equation 2 (*i.e.*, uncorrected), and Equation 9 (*i.e.*, corrected).

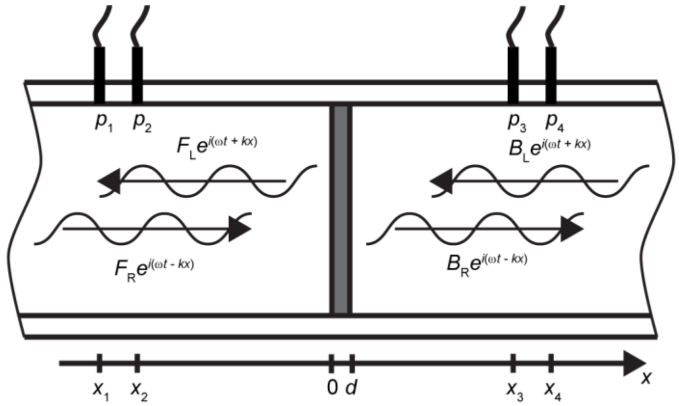

Figure S1. Acoustic waves inside the impedance tube. The membrane is placed at the center of the tube, and separates the tube into the front section and the back section. In the front section, the acoustic wave F_R travels in positive x-direction and the acoustic wave F_L travels in negative x-direction. In the back section the acoustic wave B_R travels in positive x-direction and the acoustic wave B_L travels in negative x-direction. Microphones record the pressures p_1, p_2, p_3, and p_4 at the four locations x_1, x_2, x_3, and x_4 as a function of time. With short time Fourier transformation, the signals can be decomposed into their frequency components (radial frequency ω, wave number k).

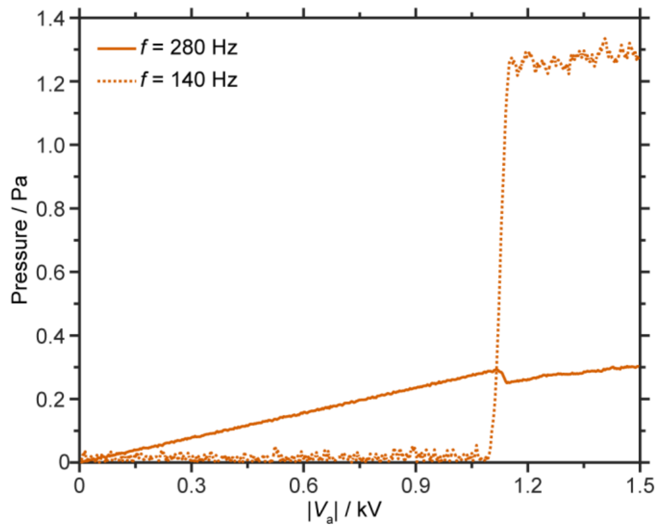

Figure S2. End of the linear regime. Fundamental (solid line) and subharmonic (dotted line) component of the acoustic spectrum of F_L as a function of $|V_a|$ at V_b = 9.0 kV, and excitation frequency f = 280 Hz. At $|V_a|$ ~ 1.1 kV a subharmonic appears in the acoustic spectrum, and the amplitude of the fundamental component drops.

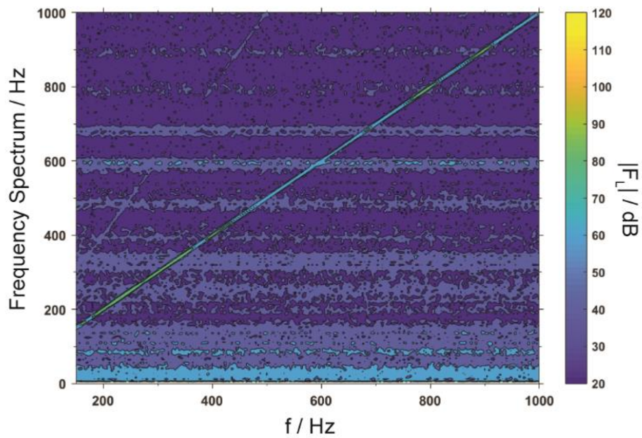

Figure S3. Absence of subharmonic. Acoustic spectrum of F_L for V_b = 9.0 kV during a sinusoidal frequency sweep from f = 150 Hz and f = 1000 Hz with amplitude V_a = 645 V. The spectrum does not contain a subharmonic.

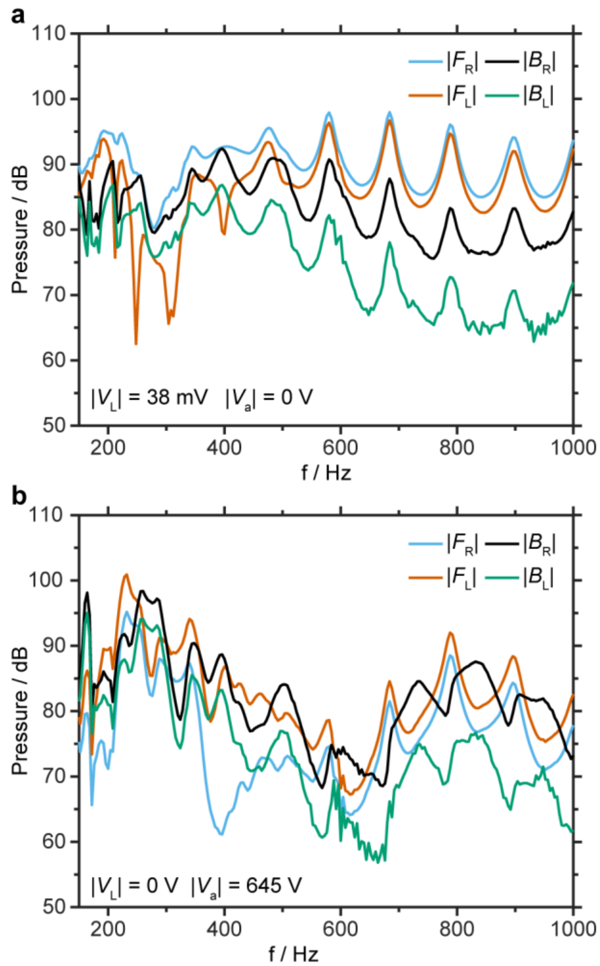

Figure S4. Characterization of the transparent membrane. Amplitudes of the fundamental component of the measured waves travelling in the front (F_R, F_L) and back (B_R, B_L) sections of the impedance tube during the sinusoidal frequency sweep on (a) the coil loudspeaker, and (b) the transparent membrane. The equidistant peaks indicate resonances of the impedance tube.

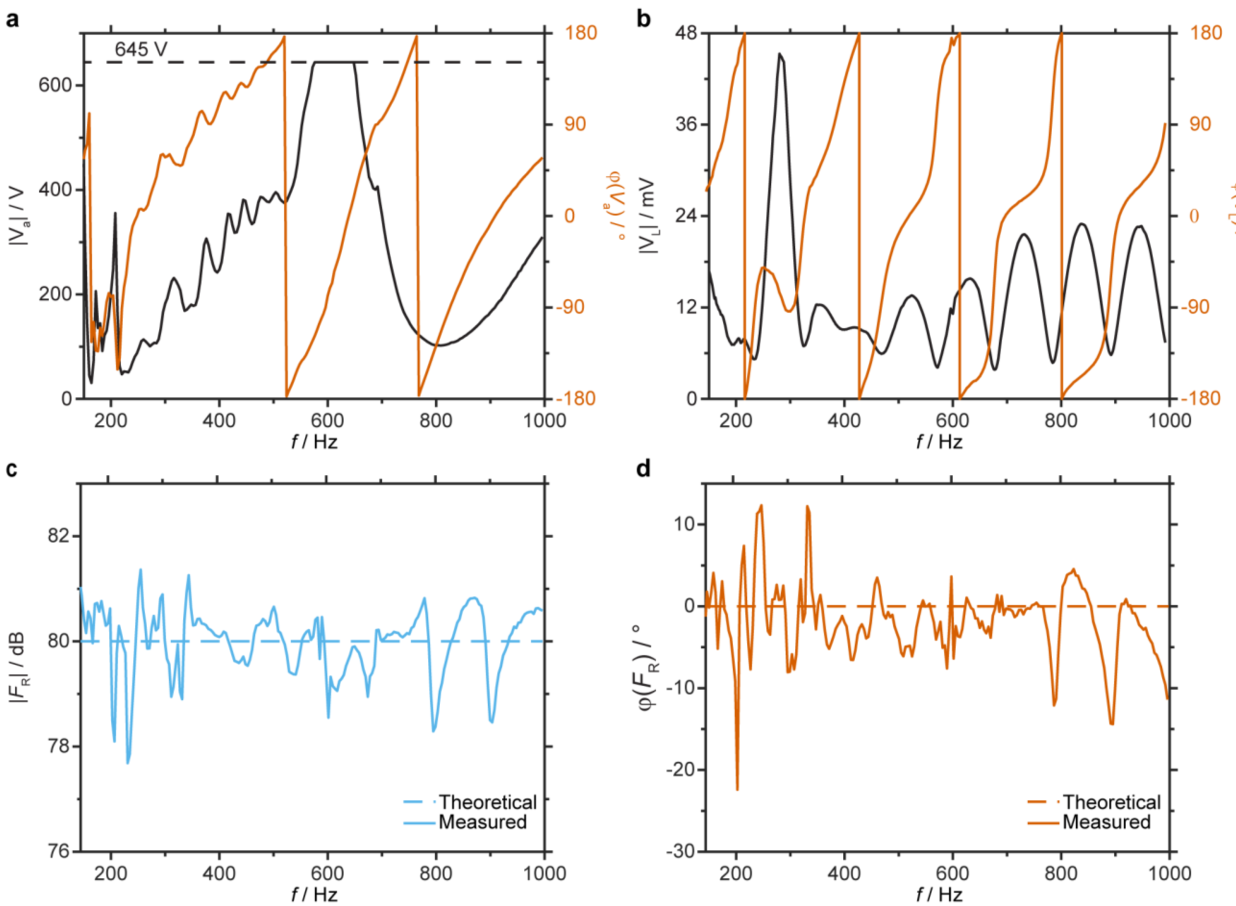

Figure S5. Active noise cancellation experiment. (a) Excitation signal applied to the transparent membrane. To ensure linearity of the membrane, the amplitude of the signal was limited to 645 V. (b) Excitation signal applied to the coil loudspeaker (c) Comparison of the amplitude of the fundamental component of the measured spectrum of F_R with the theoretically predicted amplitude. (d) Comparison of the phase angle of the fundamental component of the measured spectrum of F_R with the theoretically predicted phase angle.

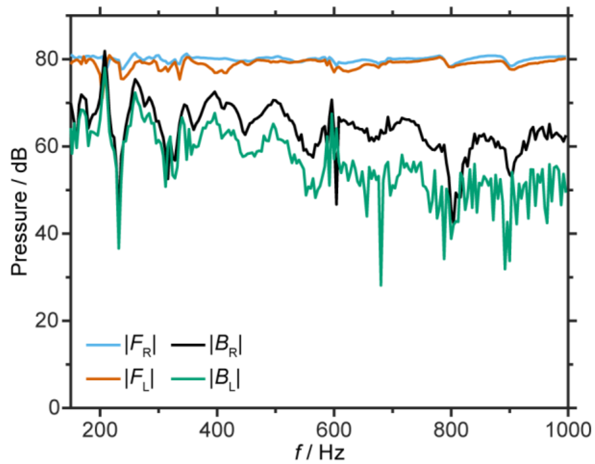

Figure S6. Fundamental components of the acoustic waves measured during the active noise cancellation experiment.

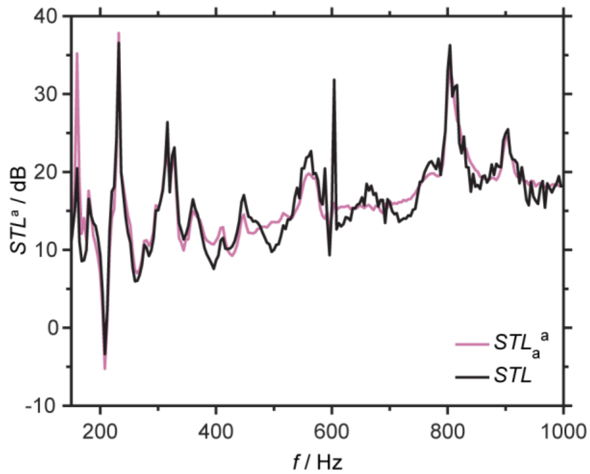

Figure S7. Comparison between the corrected STL_a^a (Equation 9) and the experimentally obtained STL (Equation 2).

References

[1] ASTM E2611 - 09: Standard Test Method for Measurement of Normal Incidence Sound Transmission of Acoustical Materials Based on the Transfer Matrix Method. doi:10.1520/E2611-09.2

[2] J. Y. Chung, D. A. Blaser, *J. Acoust. Soc. Am.* **1980**, *68*, 907.